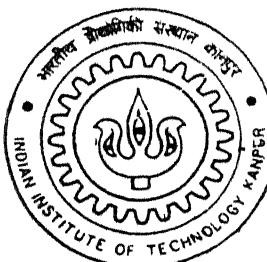


# **A Study of QoS based Management of Congestion using Backpressure Mechanism and Call Admission Control in ATM Networks**

**By**

**Vivek Mudgil**



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**DEPARTMENT OF ELECTRICAL ENGINEERING**

**Indian Institute of Technology Kanpur**

**July, 2002**

**A Study of QoS Based Management of  
Congestion using Backpressure Mechanism  
and Call Admission Control  
in ATM Networks**

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# CERTIFICATE

It is certified that the work contained in the thesis entitled *A Study of QoS based Management of Congestion using Backpressure Mechanism and Call Admission Control in ATM Networks* by Vivek Mudgil, has been carried out under our supervision and that this work has not been submitted elsewhere for a degree.

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# **SYNOPSIS**

Packet switched technology is the backbone of Internet. It provides best effort service with no prior resource allocation for two end points to communicate. By and large the design was simple and allowed efficient utilisation of bandwidth. The end-to-end congestion avoidance mechanism and reliable communication of TCP (Transmission Control Protocol) over IP was primarily intended to cater to loss sensitive traffic. With the rapid growth of Internet and improved performance of the underlying network, and the developments in convergence, there has been increasing demand for supporting delay sensitive real-time traffic over the same packet switched network. This demand for delay sensitive real time traffic is being accommodated in packet switched networks by building requisite Quality of Service (QoS) support. These services cannot be supported on the basis of best effort services and require certain Quality of Service (QoS) support from the network in terms of bounds on loss probabilities and end-to-end delay. Hence packet switching combined with the guaranteed QoS requirements of existing and new services admitted into the network have made it imperative that methods based on sound performance framework are required for managing resources and strategies for avoiding congestion in the networks. In the literature, attempts have focussed mainly on this. Our work extends the literature by taking closer look at preventing congestion by

- Blocking a newly arriving call using Call Admission Control
- Regulating the source traffic using appropriate backpressure based feedback.

Three parallel thought processes provide the motivation for this work. One is the search for a reactive mechanism that allows a higher utilisation of network resources and eliminates the possibility of sustained congestion. Second is to find a parameter that is measurable and may be used to supplement existing CAC procedures in accepting or rejecting a call. And thirdly the dependence of QoS requirements of real-time multimedia traffic on user perception, which is different from the typical packet loss requirement of traditional data networks. Hence in this work we have modelled the sojourn times (duration spent in a state) of a network element into overload and underload states. This approach has been used to study the performance behaviour of a multiplexer with

feedback-controlled sources with non-negligible propagation delay. The insight developed has been used to supplement the call admission control procedures based on asymptotic loss probability model [1].

Fluid flow approach is used to model the statistical multiplexer. The sources are modelled as On-Off type fluid sources. The combined fluid input to the fluid multiplexer is modelled as Markov Modulated Rate Process. Overload and underload periods have been defined based on two-level threshold approach. A multiplexer is in overload state between the time the contents of the fluid buffer cross the high threshold mark in upward direction and the time when the contents fall below the low threshold mark for the first time. The underload period has been defined the other way round. It is the duration from the time the contents of fluid buffer fall below the low threshold mark to the time the contents cross the high threshold mark for the first time and enter the congestion state.

The matrix partial differential equation governing the probability density of first passage time is obtained using backward Chapman-Kolmogrov equations. Laplace transform is used to obtain the solution to the matrix differential equation. This gives a solution in terms of the eigenvalues of the key matrix defined by the drift and the infinitesimal generator for the modulating Markov Process of fluid input. The most critical aspect of this problem is the correct choice of boundary conditions and the segregation of eigenvalues so as to use the eigenvalues that contribute to stability of the solution. Properties of the eigenvalues have been obtained using the approach given in [2] and [3]. These properties along with other boundary conditions have been used to obtain the solution for the probability density for first passage time to overload and underload state.

In order to obtain the sojourn times into overload and underload state, one needs to know the phases of the modulating process and the contents of fluid buffer at the instances when the multiplexer changes state. These are obtained by defining the embedded Markov Chain for the process giving phases of the modulating process at the instances of these transitions. Using the equilibrium probability of this embedded Markov chain, the explicit expressions for probability density function of sojourn times into overload and underload periods have been obtained. The numerical results have been obtained for a fluid input defined by a two state Markov Modulated Rate Process. Results have been verified with

those obtained using simulation. A Fast Simulator was developed to speed up simulation for ATM type of networks. It uses a hybrid of embedded flow equations and event based simulation to perform simulation and is especially useful for a system with large number of nodes.

The sojourn time analysis is used to obtain the overload and underload periods for three different models of fluid multiplexer with binary feedback controlled sources to study the impact of different parameters on the performance of multiplexer. The performance measures are the mean overload and underload durations. In the first model a simple rate controlled source with negligible feedback propagation delay is used. Second model uses a variation of the above model. It uses sources with source buffer. This results in the fluid multiplexer being driven by fluid input with two different modulating processes in overload and underload states.

Finally the feedback model with delayed feedback and On-Off type fluid sources with source buffer are used to obtain the sojourn times into overload and underload periods. This is used to study the impact of feedback delay, rate control on the performance on the fluid multiplexer in terms of impact on mean overload and underload periods. This model has tractability problems and has been simplified by making suitable approximations to obtain the solution. It requires obtaining state variables (state of the modulating process and contents of the fluid buffer) at the instances when the multiplexer changes state and at the instances when the modulating process changes characteristic. This happens because feedback from the multiplexer forces the On-Off sources to regulate the output of the source buffers. This changes the flow characteristic at the input of the multiplexer. This change in flow characteristic is observed after one round trip propagation delay from the instance when multiplexer changed state and sent the feedback signal.

These models have been used to obtain the mean overload and underload periods. Finally the Call Admission Control strategy is discussed. Approach extends the additive effective bandwidth mechanism proposed in [1] [4] to take into account the multiplexing gain due to statistical multiplexing of number of number of sources. The discussion suggests approaches to use the information obtained from the measurement of mean

overload and underload duration of a network element to supplement the call admission control procedures based on proposed effective bandwidth approach.

## References

- [1] Guerin, R., Ahmadi, H. and Naghshineh, M., “Equivalent Capacity and its application to bandwidth allocation in high speed network”, IEEE Journal on Selected Areas in Communications, vol. 9, No. 7, 1991, pg. 968-981
- [2] Anick, D., Mitra, D., ,and Sondhi, M.M., “Stochastic theory of a data handling system with multiple sources”, BSTJ Vol. 61, No. 8, Oct. 1982, pg. 1871-1894
- [3] Ren, Q., Kobayashi, H., “Transient solutions for the buffer behavior in statistical multiplexing”, Performance Evaluation, vol. 29, 1995, pg. 65-97
- [4] Elwalid, A. and Mitra, D., “Effective bandwidth of general Markovian traffic sources and admission control of high speed networks”, IEEE/ACM Trans. Networking, vol. 1, No. 3, 1993, pg. 329-343

**Dedicated to my wife Nandini**

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# List of Abbreviations

|        |   |
|--------|---|
| TCP    | Transmission Control Protocol                 |
| B-ISDN | Broadband Integrated Services Digital Network |
| VLSI   | Very Large Scale Integration                  |
| QoS    | Quality of Service                            |
| ATM    | Asynchronous Transfer Mode                    |
| VPI    | Virtual Path Identifier                       |
| VCI    | Virtual Circuit Identifier                    |
| CLP    | Cell Loss Probability                         |
| CAC    | Call Admission Control                        |
| UNI    | User Network Interface                        |
| ABR    | Available Bit Rate                            |
| GFR    | Guaranteed Flow Rate                          |
| RSVP   | Resource Reservation Protocol                 |
| DSCP   | Diffserv Codepoint                            |
| MMPP   | Markov Modulated Poisson Process              |
| LRD    | Long Range Dependence                         |
| VBR    | Variable Bit Rate                             |
| GOP    | Group of Picture                              |
| FCFS   | First Come First Serve                        |
| MAP    | Markovian Arrival Process                     |
| BMAP   | Batch Markovian Arrival Process               |
| IPP    | Interrupted Poisson Process                   |
| MMDP   | Markov Modulated Deterministic Process        |
| FBM    | Fractional Brownian Motion                    |
| UPC    | Usage Parameter Control                       |
| PCR    | Peak Cell Rate                                |
| CDV    | Cell Delay Variation                          |
| GCRA   | Generic Cell Rate Algorithm                   |
| EBCN   | Explicit Backward Congestion Notification     |

|      |  |
|------|--|
| EFCN | Explicit Forward Congestion Notification |
| EFCI | Explicit Forward Congestion Identifier   |
| MMRP | Markov Modulated Rate Process            |
| CTMC | Continuous Time Markov Chain             |

# List of Symbols

|                      |  |
|----------------------|--|
| $M(t), M_I(t)$       | Markov Chain modulating the flow input to a Statistical Multiplexer  |
| $M, M_I$             | Infinitesimal Generator for the Markov Chains $M(t)$ and $M_I(t)$  |
| $R$                  | Peak Rate of a single On-Off Source  |
| $R_I$                | Controlled Peak Rate of an On-Off Source   |
| $\lambda, \lambda_I$ | Transition rate out of the Off state   |
| $\mu, \mu_I$         | Transition rate out of the On State  |
| $\rho$               | Source utilization (Probability that an On-Off source is in On state)  |
| $C$                  | Link Capacity of a Statistical Multiplexer   |
| $r_i(t)$             | Input fluid flow rate at time $t$ when modulating process is in state $I$  |
| $X(t)$               | Fluid buffer contents at time $t$  |
| $X_L$                | Low Threshold of fluid buffer  |
| $X_H$                | High Threshold of fluid buffer   |
| $N$                  | Number of On-Off sources   |
| $D, D_I$             | $(N+1) \times (N+1)$ diagonal Drift matrix whose $i^{th}$ diagonal element gives the net inflow into the multiplexer with input MMRP in state $i$  |
| $p(x,t)$             | $(N+1) \times (N+1)$ matrices whose $(ij)^{th}$ elements denote the probability density function for the first passage time to crossing a threshold at time $t$ with phase of modulating process $j$ , given that the initial contents at $t = 0$ were $x$ and the phase of modulating process was $i$ |
| $\tilde{p}(x,s)$     | $(N+1) \times (N+1)$ matrices of Laplace transform of first passage density functions $p(x,t)$   |
| $z(s)$               | $(N+1) \times (N+1)$ diagonal eigenvalue matrices whose $i^{th}$ diagonal element gives the $i^{th}$ eigenvalue $z_i(s)$ of the key matrix $D^{-1}(sI - M)$  |
| $V(s)$               | $(N+1) \times (N+1)$ matrices of right eigenvectors whose $i^{th}$ column gives the right eigenvector for $i^{th}$ eigenvalue of $D^{-1}(sI - M)$  |
| $W(s)$               | $(N+1) \times (N+1)$ matrices of left eigenvectors whose $i^{th}$ row gives the left eigenvector for $i^{th}$ eigenvalue of $D^{-1}(sI - M)$   |

|                      |   |
|----------------------|---|
| $A(s)$               | $(N+1) \times (N+1)$ matrices of constants  |
| $\pi(0)$             | A row vector of size $N$ whose $i^{th}$ element gives the probability of modulating chain being in state $i$ at time $t = 0$  |
| $\pi_c$              | A row vector of size $N$ whose $i^{th}$ element gives the probability of modulating chain being in state $i$ at the embedded instant of multiplexer entering into congestion      |
| $\pi_u$              | A row vector of size $N$ whose $i^{th}$ element gives the probability of modulating chain being in state $i$ at the embedded instant of multiplexer entering into underload state |
| $P_c$                | $(N+1) \times (N+1)$ One step transition matrices from entry into congestion state at high threshold to entry into underload state at low threshold                               |
| $P_u$                | $(N+1) \times (N+1)$ One step transition matrices from entry into underload state at low threshold value to entry into overload state at high threshold                           |
| $f_c(t)$             | $(N+1) \times (N+1)$ matrices of probability density function for congestion duration   |
| $\tilde{F}_c(s)$     | Laplace of $f_c(t)$   |
| $f_u(t)$             | $(N+1) \times (N+1)$ matrices of probability density function for underload periods   |
| $L_L$                | $(N+1) \times (N+1)$ diagonal matrix with only the elements corresponding to states of MMRP that result in negative drift in the multiplexer, being 1                             |
| $L_H$                | $(N+1) \times (N+1)$ diagonal matrix with only the elements corresponding to states of MMRP that result in positive drift in the multiplexer, being 1                             |
| $B(s)$               | $(N+1) \times (N+1)$ matrix given by $V^{-1}(s)D^{-1}L_L$ and $V^{-1}(s)D^{-1}L_H$  |
| $z_A$                | Matrix whose diagonal elements are negative eigenvalues $z_i(s)$ of key matrices $D^{-1}(sI - M)$   |
| $z_B$                | Matrix whose diagonal elements are positive eigenvalues $z_i(s)$ of key matrices $D^{-1}(sI - M)$   |
| $P_1, P_2, P_3, P_4$ | Partitions $P_1(x, s), P_2(x, s), P_3(x, s)$ and $P_4(x, s)$ of matrix $\tilde{p}(x, s)$  |

|                      |   |
|----------------------|---|
| $V_1, V_2, V_3, V_4$ | Partitions $V_1(s), V_2(s), V_3(s)$ and $V_4(s)$ of right eigenvector matrix $V(s)$                                       |
| $A_1, A_2, A_3, A_4$ | Partitions $A_1(s), A_2(s), A_3(s)$ and $A_4(s)$ of constants matrix $A(s)$   |
| $B_1, B_2, B_3, B_4$ | Partitions $B_1(s), B_2(s), B_3(s)$ and $B_4(s)$ of constants matrix $B(s)$   |
| $d_f$                | Propagation delay between source and multiplexer in forward path  |
| $d_b$                | Propagation delay between source and multiplexer in reverse path  |
| $d_t$                | Roundtrip propagation delay between source and multiplexer  |
| $Y_i$                | Random variable denoting size of fluid impulse when MMRP is in state $i$  |
| $T_n$                | Random instants when Markov Process modulating the input flow changes its state   |
| $Z_n$                | Random variable denoting the net fluid that gets added or removed in the sojourn period $[T_{n-1}, T_n)$                  |
| $g_i(z)$             | Probability density function denoting the potential net fluid inflow or outflow in a sojourn period in a particular state |
| $\alpha_{i-1}^i$     | Denotes the probability that given the state of MMRP to be $i$ , the previous state was $i-1$                             |
| $\alpha_{i+1}^i$     | Denotes the probability that given the state of MMRP to be $i$ , the previous state was $i+1$                             |
| $C_E$                | Exact effective bandwidth of N sources  |
| $\hat{C}_N$          | Estimate of effective bandwidth of N sources  |

# **Chapter 1**

## **Introduction**

In last few years the growth of Internet has been supplemented by the ever-increasing demand for new services. One of the keys to this success of the Internet has been the congestion avoidance mechanism of TCP (Transmission Control Protocol). The end-to-end congestion control mechanism of TCP has scaled well with the increase in Internet traffic. But this growth has also resulted in an increase in number of applications that do not use TCP e.g. multimedia streaming, multicast etc. Since these applications are not controlled by any end-to-end congestion avoidance mechanism like that of TCP, there exists a potential for future congestion collapse of the Internet [FLO00]. Hence there is a need for mechanisms for these traffic classes so as to prevent and react to congestion in the network. This work studies the problem of congestion control by modeling the sojourn time behaviour of a network node into congested and uncongested states. This is subsequently used to recommend appropriate call admission control procedure to regulate the traffic entering the network, thereby reducing both the possibility and duration of congestion.

Internet was primarily designed for data communication. It is based on packet switching and it provides the best effort delivery. There is no fixed bandwidth allocated to a service. The available bandwidth is shared amongst all the user traffic streams. This multiplexing of different streams in statistical manner results in efficient utilization of prime network resources [KLE][BER87]. Congestion is avoided by restricting the traffic in the network through end-to-end flow control and packet discarding mechanism of TCP.

Today Internet supports data, voice and video communication in a single network. This has been made possible by advances in audio, video and speech coding and compression algorithms and progress in VLSI technology, resulting in significantly reduced bit-rate generated by these services. This has created a demand for services that were otherwise prohibitive because of their high bandwidth requirements. These services have dynamically changing bandwidth requirements and tend to be highly bursty in nature. While some of these services are loss sensitive, others have Quality of Service (QoS) requirement in form of bounded delay guarantees. Hence the traffic

behavior and QoS required to be met by the underlying carrier network turn out to be complex [ONV94] One or more of the following attributes may characterize these new services

- High bandwidth
- Bandwidth on demand
- Varying quality of service parameters
- Guaranteed Service Requirements
- Point-to-point, point-to-multipoint or multipoint-to-multipoint connection
- Continuous or Variable bit rate service
- Connection oriented or connectionless service

Thus, it is not sufficient for today's network to follow the best-effort delivery model of the Internet, but also support QoS guarantees Considering the delay sensitive nature of important real-time applications like voice and video, it needs to be ensured that the network is able to support applicable QoS guarantees This in turn has fundamental implication in the way traffic management across the network is implemented Traffic Management constitutes management of time and space resources in the form of priorities and includes procedures to restrict the traffic in the network through preventive measures like call blocking [BRA94]

In the last one-decade, there have been two parallel developments to provide integrated services in a single network One is the development of integrated broadband data network and another is the effort to integrate new services over the ubiquitous Internet It was envisaged that a Broadband Integrated Services Data Network (B-ISDN) would be a single network with basic communication capabilities to support both existing as well as new applications and services Other was a parallel effort in the direction of developing mechanisms so that the Internet routers are able to reserve resources and provide service specific QoS This requires flow-specific states in the routers, which represent a fundamental change to the existing Internet model [BRA 94] of end-to-end flow control These two parallel developments have tried to address the issue of integrating different types of services in one single, QoS enabled network The two developments are discussed next in Section 1 1 - 1 3 describing the mechanisms to provide QoS guarantees

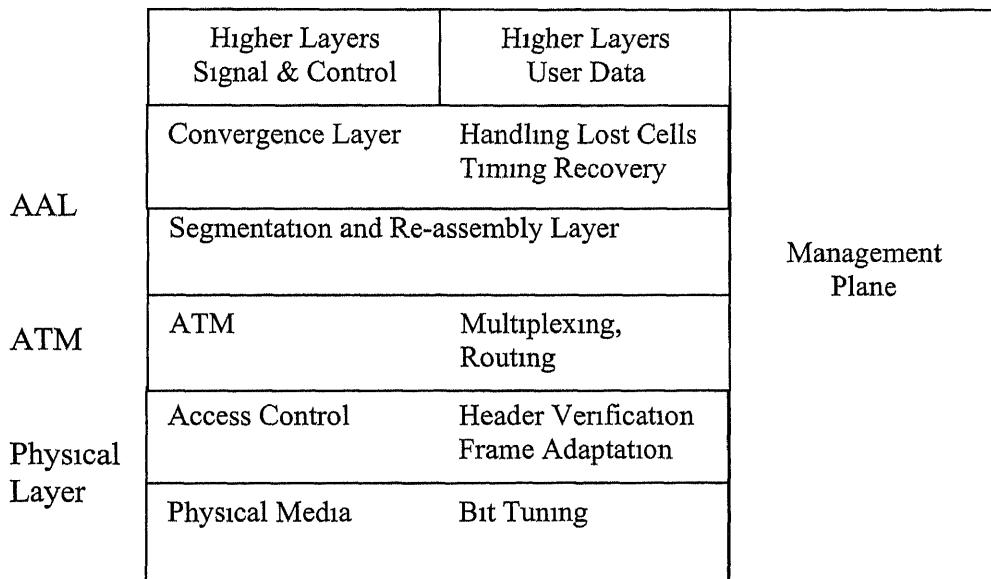
## 1.1 B-ISDN

Broadband Integrated Services Digital Network (B-ISDN) was conceptualized keeping in view the desirability and requirements of single network for all types of services. The concepts of B-ISDN are summarized in CCITT Recommendations I 121

*“B-ISDN supports switched, semi-permanent, point-to-point and point-to-multipoint connection and provides on-demand, reserved and permanent services. Connections in B-ISDN support both circuit mode and packet mode services of a mono and multimedia type and of a connection-less or connection-oriented nature and in bi-directional and unidirectional configuration. A B-ISDN will contain intelligent capabilities for the purpose of providing service characteristics, supporting powerful operation and maintenance tools, network control, and management”*

In short, the objective of B-ISDN is to provide

- A single, standard user/network interface for all services
- Support for existing and future services with varying characteristics
- QoS guarantees as required by each individual service
- Unified traffic management procedures



**Figure 1.1 B-ISDN Protocol Reference Model**

The B-ISDN protocol reference model (Figure 1.1) defines three-layered architecture consisting of Adaptation layer, Transfer Mode Layer and Physical Layer

- Physical Layer for B-ISDN supports various transmission media based on different standards like SONET, DS3 etc It transfers packets between two B-ISDN entities
- Transfer Mode Layer based on Asynchronous Transfer Mode (ATM) [Section 1.2] provides packet multiplexing, de-multiplexing and routing functions It also handles traffic management functions like call establishment and policing so as to provide Quality of Service guarantees to all the streams The asynchronous nature of the transport mechanism is the most important aspect of B-ISDN
- Adaptation Layer supports higher-layer user and control functions of the various services These include functions like segmentation and re-assembly, detection and action on lost cells, padding, handling cell delay variations etc

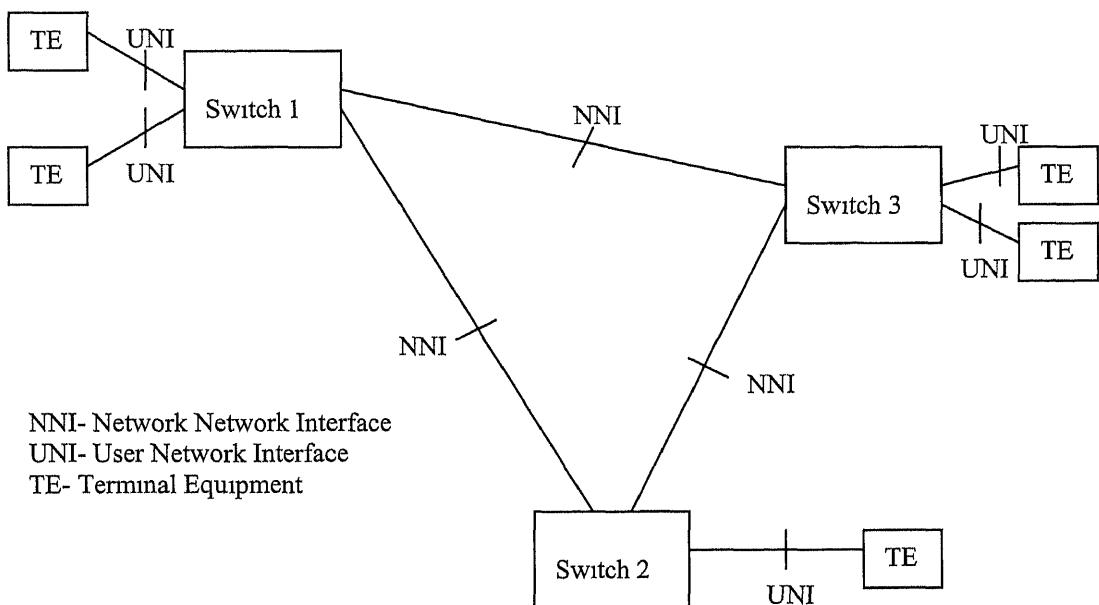
## 1.2 Asynchronous Transfer Mode

The ATM layer provides the QoS support in B-ISDN Asynchronous Transfer Mode [GON86] is a fast packet switching and multiplexing technology that combines the benefits of circuit switching (guaranteed capacity and constant transmission delay) with those of packet switching (flexibility and efficiency for bursty traffic) It provides scalable bandwidth from few megabits per second to many gigabits per second ATM promises technical capability to handle any kind of information, voice, data, image, text and video in an integrated manner It has been designed to carry various traffic types while satisfying the QoS guarantees for each of the traffic types Hence it has been chosen as a proposed telecommunication standard for B-ISDN and networks based on ATM are being deployed throughout the information infrastructure especially in carrier backbones

ATM is different from synchronous technologies in the following way In case of synchronous technologies such as time division multiplexing, each user is assigned a time slot, and no other user can use that time slot A source can only send data in its time slot even if the other time slots are empty In case of ATM, its asynchronous nature allows for the availability of time slots on demand This leads to efficient multiplexing of streams with bursty traffic resulting in higher utilization of bandwidth

This flexible availability of bandwidth is not possible with synchronous technologies. Thus the asynchronous mode of transfer inherits the advantage associated with packet switching.

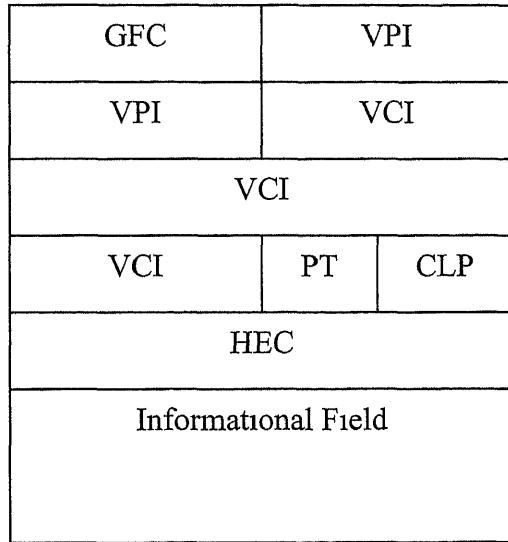
In packet switched networks, packets have variable lengths. This requires complex buffer management inside the network, increasing the complexity of a network element. Secondly the service time delay associated with each packet is also proportional to the size of the packet. This results in variable transmission delay for each packet. This is addressed in ATM as it switches small-sized (53 bytes) packets called cells. While small cell size results in reduced service delay, the fixed size of the ATM cell results in almost constant transmission delay. Though the disadvantage of small size of ATM cells is the increased overhead of cell header.



**Figure 1.2 ATM Network**

An ATM network is made of ATM switches and ATM end-points (Figure 1.2). ATM is a connection-oriented technique, requiring establishment of end-to-end connections before the actual transfer of information between two users can take place. A connection is established only if each intermediate switch in the route has resources available to support QoS requirement for the connection. An ATM switch is responsible for cell transit through an ATM network. It accepts the incoming cell from an ATM end-point or another ATM switch. It then reads the routing field in the cell header of an incoming cell, performs a table lookup to determine the outgoing link and

updates the cell header with the new routing identifiers. The cell with updated header is then quickly switched to the corresponding interface towards its destination. The table entries at each node are written at the connection establishment phase for each connection. Thus the intermediate nodes perform the function of cell relaying. The simple routing due to connection-oriented technology in ATM allows fast switching of packets, thus reducing end-to-end delay. It allows ATM to scale with the increasing bandwidths of the underlying physical layers.



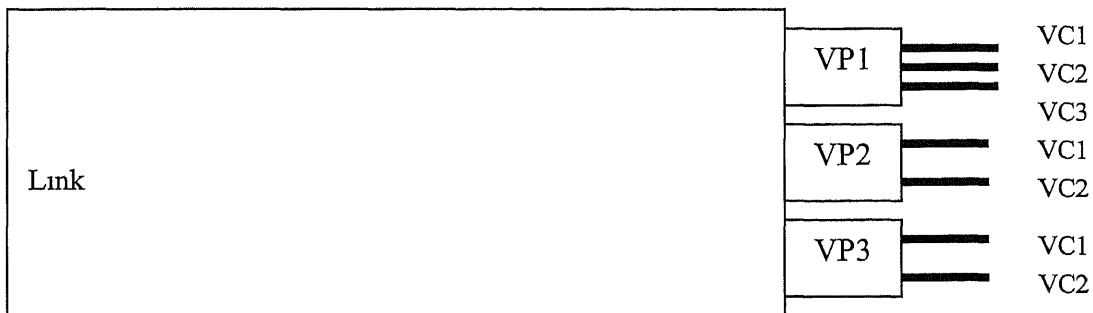
**Figure 1.3 UNI Cell Format**

### 1.2.1 ATM Cell Structure

ATM cell structure has been defined such that each cell carries information that is used by intermediate switches to decide on priority based switching and discarding of each cell. This allows the switches to provide connection specific QoS guarantees. A 53-byte ATM cell has 5 bytes for header and 48 bytes for information payloads. As shown in Figure 1.3, the ATM header contains the following fields:

- Generic Flow Control (GFC)
- Virtual Path Identifier (VPI)
- Virtual Channel Identifier (VCI)
- Payload Type (PT)
- Cell Loss Priority (CLP)
- Header Error Control (HEC)

Generic Flow Control is a four-bit field providing flow control for implementing multi-access schemes like DQDB over ATM. The ATM provides for two level routing hierarchies that are based on virtual paths and virtual channels. VPI and VCI fields together provide identification for a connection. The Payload Type field is used to indicate the kind of information a cell is carrying, such as user information or information for operation and maintenance. The CLP field allows for two level cell loss priority. Due to the statistical multiplexing of connection, it is unavoidable that cell losses will occur in a B-ISDN. A cell with the CLP bit set may be discarded by the network during congestion, whereas cells with the CLP bit cleared have higher priority and shall not be discarded if at all possible. The Header Error Control field is used for two purposes to discard cells with corrupted headers and for cell delineation.



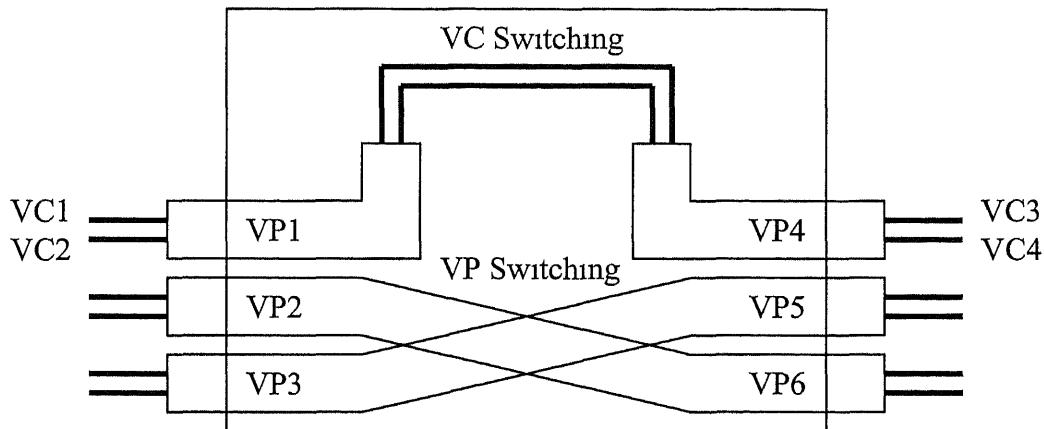
**Figure 1.4 Virtual Paths and Virtual Circuits**

### 1.2.2 ATM Switching

The packet switching mechanism of ATM provides efficient utilization of bandwidth in the network. The routing in ATM network is based on Virtual Path (VP) and Virtual Channel (VC) concept. Virtual Channel is a concept used to describe unidirectional transport of ATM cells associated by a common Virtual Circuit Identifier. After finding a path between the two end nodes, the network assigns the VCI values to be used at each link along the path and sets up the routing tables at each intermediate node. Upon completion of the communication, one of the end users releases the connection by deleting the corresponding entries from the routing tables at each intermediate node. A cell on the incoming link is switched by obtaining the outgoing port and new VCI from the routing table.

The concept of Virtual Path is introduced to reduce the size of routing tables by bundling a number of virtual circuits that use more than one common links between

two nodes in a network (Figure 1.4) Thus a VP is a logical direct link between two nodes in the network that are connected via two or more sequential links The cells belonging to different virtual circuits are switched using the common VPI along the nodes of the VP, reducing the routing table size In VP switching, VCs multiplexed into VPs are switched from the input to the output links using only the VPI field of cell headers VCIs of connections passing through VP switches remain unchanged and have no significance in routing cells between input and output ports of a VP switch Figure 1.5 shows the VP/VC switching



**Figure 1.5 VP/VC Switching**

### 1.2.3 ATM Services

ATM provides three types of services, Permanent Virtual Connection, Switched Virtual Connection and Connectionless service Permanent Virtual Connection (PVC) guarantees availability of a connection and does not require call setup procedures between switches Disadvantage of PVC is that it requires static connectivity and manual setup Each piece of equipment between the source and the destination must be manually provisioned for the PVC

A SVC is created and released dynamically and remains in use only as long as data is being transferred In this sense, it is similar to a telephone call The advantages of SVC include connection flexibility and call setup that can be handled automatically by a network device In a connectionless service, the cells belonging to different sources are transported using same VPI/VCI over the already established connection between two end nodes

## 1.2.4 Traffic Control

Traffic control is a critical component for enabling QoS in ATM Networks. The concept of traffic control in ATM networks is very simple. A user setting up a connection specifies the traffic characteristics of a cell stream and if the network judges that enough resources are available, it accepts the connection. After acceptance, the cell stream from the connection is monitored to assure conformance between the actual cell stream and the value specified at the setup.

Though the concept of traffic control is simple, its implementation poses serious challenges. The need to cope with statistical behavior of multiplexer makes the problem difficult. The diversity of traffic characteristics and QoS requirements further complicate the issue.

The traffic management functions at each intermediate node on the path of the requested connection, assign the necessary resources. If the resources are not available, the connection is rejected. The traffic management functions at the edge of the network monitor the conformity between the declared traffic characteristics and the characteristics of the actual cell stream at the entrance of the network [ATM99]. Traffic Management Specification [ATM99] of ATM Forum specifies six service categories. For each category, a set of traffic descriptors is specified to describe the characteristic of traffic presented to the network, and the Quality of Service that is required of the network. ATM Forum specification defines a number of traffic control mechanisms for providing the specified quality of service.

- Connection Admission Control
- Usage Parameter Control
- Selective Cell Discard
- Traffic Shaping
- Explicit Forward Congestion Indication
- Resource Management using Virtual Paths
- Frame Discard
- Generic Flow Control
- ABR Flow Control

The following QoS parameters are negotiated between the end-systems and the network at the time of establishment of connection:

- Peak-to-peak Cell Delay Variation (peak-to-peak CDV)

- Maximum Cell Transfer Delay (max CTD)
- Cell Loss Ratio (CLR)

The various mechanisms that have been proposed for implementing the Traffic Management for ATM networks are discussed next. Call Admission Control (CAC) function is defined in [ATM99] as the set of actions taken by the network at Switched Virtual Connection establishment or by Network Management during Permanent Virtual Circuit (PVC) establishment in order to determine whether a connection can be accepted or should be rejected. The CAC algorithm to make this decision uses the traffic description provided by the users at connection setup time. Various algorithms for CAC are discussed in Section 2.3.1

Usage Parameter Control (UPC) functions perform the function of policing the traffic on Virtual Connections (VCs) and Virtual Paths (VPs). Its main purpose is to monitor and enforce the traffic contract. The different UPC mechanisms proposed in the literature are discussed in Section 2.3.2.

Selective Cell Discard function defines the strategy for discarding cells when a network element faces congestion. The ATM Cell header format allows the use of two level priorities defining a cell to be of higher priority or lower priority. Two mechanisms have received considerable attention in the literature: push out and threshold. In this mechanism the low priority cell is discarded if the buffer is full but a high priority cell is only discarded if there is no low priority cell in the buffer. Threshold based mechanism defines a threshold and when the queue size exceeds this threshold value, all low priority cells are discarded until the queue size falls below the threshold.

Traffic shaping is a mechanism that alters the traffic characteristics of a stream of cells on a connection to achieve better network efficiency whilst meting the QoS objectives, or to ensure conformance at a subsequent interface [ATM99]. The peak cell rate and the mean cell rate bound the amount of bandwidth reserved for a connection. The reduction in peak rate of a cell stream can result in significant saving of bandwidth but this has an associated cost in terms of increase in the delay. The traffic shaping may be performed at the source or at the UNI or within the network. The implementation is network specific and it requires introduction of a buffer in a cell stream with output rate set to the targeted peak rate of the stream.

Resource management using Virtual Paths allows simplification of CAC, segregates Virtual Connections according to the service category, improves efficiency

in signaling as single message has to be sent for all VCs in a VP, and allows UPC to be applied to aggregate traffic [ATM99].

Frame discard mechanism allows the discard of all cells that constitute a frame or higher layer protocol data unit. This is possible whenever frame boundaries can be delineated in the SDU-type field header of an ATM cell. In GFR service category the frame discard is enabled. In other categories it is network specific.

The use of Generic Flow Control at the UNI has not been defined in ATM Forum Traffic Management Specification [ATM99]. This function is specified in ITU-T Recommendation I.150 and ITU-T Recommendation I.361

ABR flow control occurs between a sending end-system (source) and a receiving end-system (destination). A source generates forward Resource Management (RM) cells which are turned around by the destination and sent back to the source as backward RM-cells. These backward RM-cells carry feedback information provided by the network elements and/or the destination back to the source. A network element may directly insert feedback control information into RM cells, indirectly inform the source about congestion by setting EFCI (Explicit Forward Congestion Indication) bit in the data header of cells of the forward flow or generate backward RM-cells [ATM99]

## 1.3 Internet QoS

The Internet architecture has been founded on the concept that all flow-related state should be in the end-system [CLA88]. It is this design concept, which has provided robustness to the TCP/IP protocol suite and is the key to its success. It has been suggested in [BRA94] that for integrating real-time services in the Internet, routers must implement an appropriate QoS for each flow. Work on QoS enabled IP network has led to two different approaches. One is based on Integrated Services Architecture (Intserv) [BRA94] and uses Resource Reservation Protocol (RSVP) [BRA97] for providing QoS to each flow. The other approach is based on the Differentiated Services Architecture (Diffserv) [BLA98].

The Integrated Services Architecture (Intserv) [BRA94] defined a set of extensions to the traditional best effort model of the Internet with the goal of allowing end-to-end QOS to be provided to applications. One of the key components of the architecture is a set of service definitions. The current set of services consists of the controlled load and guaranteed services. The architecture assumes that some explicit setup mechanism is used to convey information to routers so that they can provide

requested services to flows that require them. While RSVP is the most widely known example of such a setup mechanism, the Intserv architecture is designed to accommodate other mechanisms.

RSVP is a signaling protocol that applications may use to request resources from the network. The network responds by explicitly admitting or rejecting RSVP requests. Intserv defines the models for expressing service types, quantifying resource requirements and for determining the availability of the requested resources at relevant network elements (admission control). RSVP signals per-flow resource requirements to network elements, using Intserv parameters. These network elements apply Intserv admission control to signaled requests. In addition, traffic control mechanisms on the network element are configured to ensure that each admitted flow receives the service requested in strict isolation from other traffic. To this end, RSVP signaling configures microflow (MF) [BLA98] packet classifiers in Intserv capable routers along the path of the traffic flow. These classifiers enable per-flow classification of packets based on IP addresses and port numbers. The necessary policy control mechanisms like access control, authentication, and accounting have been defined in [BOY00].

In contrast to the per-flow orientation of RSVP, Diffserv networks classify packets into one of a small number of aggregated flows or "classes", based on the Diffserv codepoint (DSCP) in the packet's IP header. This is known as behavior aggregate (BA) classification [BLA98]. At each Diffserv router, packets are subjected to a "per-hop behavior" (PHB), which is invoked by the DSCP. The primary benefit of Diffserv is its scalability. Diffserv eliminates the need for per-flow state and per-flow processing and therefore scales well to large networks.

## 1.4 Motivation & Organisation of the Thesis

The bursty nature of traffic in broadband networks requires statistical multiplexing for efficient utilization of resources. This results in increased loss probability as more and more streams are allowed into the network. Hence one of the method to restrict traffic into the network and provide guaranteed bound for cell loss probability in broadband networks is to reserve the resources and reject the admission of a stream into the network if no resources are available. The problem of resource allocation in a statistical multiplexer has been addressed based on steady state analysis of the multiplexer queue. These do not take into account the transient behavior of the queue. Hence even with provisioning of appropriate resources there is a good

possibility of stream experiencing poor QoS because of transient behavior of the queue. The distribution of overload periods has a direct impact on the quality of service (QoS) perceived by the end-user. Hence sojourn time based metrics do play a crucial role in the effectiveness of various congestion control schemes in multimedia broadband networks. [RAM95], [TIP93], [SIM92].

The Call Admission Control (CAC) procedures control the traffic entering the network by restricting the number of connections. Such a preventive approach reduces the probability of congestion but still cannot eliminate the possibility of sustained congestion occurring within the network. This requires some kind of reactive measure to control the congestion from sustaining for longer duration.

Motivation for this work came from three factors. One is the search for a reactive mechanism that allows higher utilisation of network resources and eliminates the possibility of sustained congestion. Second is to find a parameter that is measurable and may be used to supplement existing CAC procedures in accepting or rejecting a call. And thirdly the dependence of QoS requirements of real-time multimedia traffic on user perception, which is different from the typical packet loss requirement of traditional data networks. In this thesis the sojourn period has been modeled using fluid flow approach. The duration and periodicity of congestion reflects how heavily a node is loaded. Hence this may be used as one of the parameter in taking the decision of admitting a new call. The sojourn time analysis has been extended to obtain the moments of the distribution and periodicity of overload period for multiplexer with feedback controlled sources. This is used to study how the reactive mechanism based on simple binary feedback effects the mean congestion duration and its periodicity. Though the main focus of the work in this thesis has been on ATM Networks, it may be extended to general packet multiplexer like Internet router.

Chapter 2 gives a survey of issues involved in modeling of an ATM multiplexer. The issue of traffic modeling and its usage in analytical study of multiplexer behavior has been discussed. The advantages and limitations of each have been listed. The discussion also covers the issues involved in management of traffic in the ATM networks with diverse traffic characteristics and QoS requirements.

In Chapter 3 the usage of fluid model has been justified. This is followed by a simple queue modeling with deterministic server to obtain the distribution of the sojourn period. The boundary conditions for the system of differential equations defining the distribution for the overload period and under-load period have been

defined. These conditions are used to obtain the moments of congestion duration and the duration for which the multiplexer remains in uncongested state in Chapter 4.

In Chapter 4 the approximate two-state model is used for the superposed traffic from N On-Off sources to obtain the distribution for the congestion period using the results obtained in Chapter 3 and 4. This is compared with the results obtained from simulation. Chapter 4 also discusses the simulation methodology used. A fast simulator using embedded flow equations developed for the purpose of simulating network of switches has been used for simulation.

In Chapter 5 the congestion duration is modeled for a multiplexer with rate controlled sources. The binary feedback mechanism controls the output rate of the source buffer. This model has been used to study the congestion duration and other parameters under non-negligible propagation delay. The sojourn time analysis developed in Chapter 3 and 4 has been extended to obtain the distribution for the duration of congestion. The performance of feedback mechanism has been studied using models with and without source buffer, finite and infinite multiplexer buffer and negligible and non-negligible propagation delay. This extension may be used to model the back-pressure mechanism of flow control.

Chapter 6 discusses the call admission control strategy proposed for open loop congestion control. The basic objective of a bandwidth management and traffic control strategy is to allow for high utilization of network resources, while sustaining an acceptable grade of service for all connections. In this chapter a call admission control based on equivalent capacity obtained from the asymptotic approximation, has been proposed. This methodology is based on fluid flow analysis of a single server queue, served by traffic that is represented as deterministic flow with random jumps. The approach permits the calculation of asymptotic constant for the buffer occupancy distribution, iteratively. This leads to more realistic calculation of equivalent capacity than those based on additive effective bandwidth. Another approach based on calculation of asymptotic constant using the instant traffic rate as a continuous random variable has been proposed and results obtained are found to be very close to those obtained by the first method. The results obtained from theoretical modeling have been compared with those from computer simulation and through exact computations.

Chapter 7 summarizes the results and discusses the scope of future work.

## **1.5 Summary**

The need for QoS enabled Internet has led to the development of ATM network on one hand and mechanisms that provide QoS support to the existing Internet on the other. Various aspects of ATM network that contribute to QoS guarantees have been discussed here. The efforts to provide QoS support to Internet through Intserv and Diffserv have been described. The Chapter concludes with the motivation for the work done in this thesis.

# **Chapter 2**

## **Literature Survey**

Traffic management in communication networks deals with the controlled use of network resources to prevent congestion in the network while providing the negotiated Quality of Service (QoS) to the users. The primary objective of traffic management is efficient utilization of resources especially in packet switched networks where the statistical nature of multiplexing implies that there is no dedicated allocation of bandwidth as in the circuit switched networks. New issues in traffic management have come up with the demand for support of different kind of services like voice, video and data traffic in an integrated manner. Successful traffic management is the key to the deployment and management of high-speed broadband networks.

Broadband networks support interactive and distributive services, bursty and continuous traffic, connection oriented and connectionless services, point to point, point to multipoint and multipoint to multipoint communications in the same network [Section 1.1]. Each service has different service requirement, some are delay sensitive, and some are loss sensitive while some are sensitive to both. As mentioned in Section 1.2, ATM networks provide a functional framework for supporting these services and have been accommodated as a transport layer for B-ISDN by ITU-T.

A broadband network is made up of switches and routers connected through links with output/input buffers. Each of these switches perform traffic management functions in order to provide QoS support to the supported connections. While the QoS guarantees in an uncongested network node can be ascertained by well-designed traffic management functions, it is during the congestion duration that negotiated QoS need to be guaranteed. Even with the methods proposed for traffic management at each switch/router, congestion occurs even with increase in buffer size, bandwidth and processing power [LIE95], [GUS90]. The duration of congestion is a key parameter in determining the QoS guarantees at a switching node and hence the end-to-end QoS guarantees. Controlling the flow input to the congested node can control congestion at any node. In TCP/IP networks the flow control is end to end. It does not scale well with heterogeneous traffic and service requirements, and increased bandwidth-propagation delay [DIG91], [MIS92], [WAN91] of the broadband networks. Studies have shown

that closely coupled network elements using explicit feedback perform better in reducing the overload periods in packet switched networks [MIS92] [RAM93] [MUD95A].

This chapter provides an insight in to the traffic management policies proposed in the literature, bringing out the advantages and limitations of different methods. For this, it is necessary to comprehend the characteristics and requirements of the traffic to be carried. Traffic modeling and teletraffic analysis are basic to the modeling, analysis and evaluation of networks. This chapter reviews the different traffic models, highlighting their effectiveness and limitations to model different sources. ATM network itself is viewed as a collection of queues connected in a manner determined by the network topology. Each link in an ATM network has an associated buffer. Thus, a link is viewed as a server with associated buffer queue.

A well-known approach in queuing theory to analyze large queuing networks is to decompose the network into individual queues and analyze each queue in isolation. Thus the study of ATM networks may be reduced to study of individual queues in the network. Modeling the traffic arriving at a switch and then designing a packet multiplexer has been an important area of study in ATM networks [ONV94]. Some of the methodologies used to model and analyze a typical queue for ATM networks are discussed in this chapter. This is followed by a discussion on various components of traffic management. Section 2.4 discusses the various feedback models proposed in the literature. We then state the problem that has been examined in detail in this thesis.

## 2.1 Source Characterization

Traditional teletraffic analysis has frequently used the assumption that the packet inter-arrival time is exponentially distributed. The Poisson model is reasonably good for modeling data sources and is especially good for modeling the traffic from a large number of sources. In the context of ATM networks, there was a requirement for adequate source models that will take care of bursty nature of traffic generated by different source types such as file transfer, compressed video and compressed voice with silence suppression. The new models for the traffic arrival process at a network node need to capture this burstiness in the traffic. Such a model should not only reasonably model the measured characteristic of a source but should also be analytically tractable for meaningful usage in teletraffic analysis. This section deals with the characteristics of different sources and some of the models proposed for the

traffic generated by these sources. These models characterize statistical traffic in terms of parameters such as burstiness, inter-arrival time, inter-arrival correlation etc.

### 2.1.1 Video Sources

Traffic characteristic of a video source depends largely on two main factors namely scene contents and coding algorithms. Hence exact characterization of traffic is very difficult. During every frame interval cells are generated at a peak rate (this again depends on the codec). The duration of a burst depends on the information content in the frame. It has been shown [KIS89] that the packet generation interval has a complex combination of different distributions. Video traffic can be classified in to two categories based on uniform and non-uniform activity levels.

Uniform-activity video constitutes low bit rate video (like video telephone) where the change in the scene is not significant. Video traffic from such sources shows a short duration correlation, which decay exponentially with respect to the time. It has been shown [NOM89] by simulation that the average and distribution of a queue of a statistical multiplexer with infinite buffers are well approximated with the first-order autoregressive model. In autoregressive approach, a video bit rate is modeled as a weighted sum of finite number of previous bit rates. The first-order autoregressive model proposed in [MAG88] estimates the bit rate at the  $n^{th}$  frame from the bit rate at the  $(n-1)^{th}$  frame.

If  $\lambda(n)$  is the bit rate of the  $n^{th}$  frame, then

$$\lambda(n) = a\lambda(n-1) + b\omega(n) \quad (2.1)$$

where  $a, b$  are constants and  $\omega(n)$  is a Gaussian random variable. The constants  $a$  and  $b$  are obtained from the mean and autocovariance of the measured bit rate of video traffic. This model has been found to be fairly accurate [MAG88] but is not suitable for analytical studies of queuing models and is mostly used for simulations. Autoregressive model has also been used in [HEY92] [RAM92] for modeling video traffic based on observed autocorrelation function of teleconference video.

An analytically tractable model of a variable bit rate video source is a Markov Modulated Rate Process [MAG88][ELW93]. In this approach a video stream is modeled as having a fixed rate  $\lambda(i)$  when the source is in state  $i$  and is modulated by the discrete state Markov process with  $N$  states [ELW93]. Larger value of  $N$  increases the accuracy of the model [MAG88] but at the same time increases the complexity of

models used to analyze systems with such sources. The parameters of the model are obtained by matching the mean, variance and autocovariance function of traffic rate obtained from the Markov process with that obtained from measurements. While this model is good for capturing the correlation in the traffic stream, it cannot take care of periodicities. This model tends to be inaccurate for use in modeling queues with small buffer size [NAG91]. This is because it does not take into consideration the variation in cell inter-arrival time.

Markov Modulated Poisson Process [FIS93] is a doubly stochastic Poisson process. The arrivals occur in a Poisson manner with the mean rate of arrival  $\lambda(i)$  when the modulating Markov Process is in state  $i$ . The state of the arrival process is determined by the state of a modulating  $N$  state Markov process, which is independent of the arrival process. It has the property of capturing both the time-varying arrival rates and correlations between the interarrival times. The parameters for the MMPP may be obtained by matching the statistical characteristic of the MMPP process with those obtained from measurements. Approximation techniques proposed in [NAG91] [HEF96] [BAI91] [RAM91] may be used for obtaining the MMPP parameters. Though the numerical procedures using MMPP model have the disadvantage of slow convergence at high traffic intensities [NEU81], it has been widely used for the analysis of ATM networks [NAG91] [FIS93] [RAM91] [CHE93] [SAI91] [SAI91A].

Although the Markovian models are able to capture the fast decaying short term correlation in the video stream, the long term correlation in the scene changes present in the non-uniform activity video cannot be captured using these models. This class of video shows long-range dependence (LRD) in video traffic. Long-range dependence and heavy-tailed marginals in video traffic are discussed in [BER95] [GAR94] [JEL97] [HEY96]. The long-term slow decaying correlation has been captured in [SEN89] by use of two dimensional, continuous-time Markov models. It has been shown in [GAR94] that the tail behavior of the marginal bandwidth distribution can be accurately described using 'heavy-tailed' distributions (e.g., Pareto) and the autocorrelation of the VBR video sequence decays hyperbolically (equivalent to long-range dependence) and can be modeled using self-similar processes [LEL94]. Self-similar processes are attractive alternatives to traditional queuing models because they require few parameters to describe the complex packet traffic process. But models of systems using

such an arrival process are analytically less tractable. Thermodynamic entropy based model [DUF95] have also been reported in the literature.

MPEG video is composed of a cyclic sequence of frames, called Group of Picture (GOP), which contains an initial  $I$  frame and some  $P$  and  $B$  frames. Most of MPEG video streams have  $I$  frames much larger than  $P$  and  $B$  frames. This means that the information burst occur at the beginning of each GOP. A discrete-time variant of semi-Markov process with independent, identically distributed (i.i.d.) GOP bit rates in each renewal interval has been used to model MPEG video [JEL97]. A modeling framework based on the extension of transform-expand-sample (TES) processes, capturing the marginal density and correlation structure of traffic, has been proposed in [MEL98]. This model is good for creating high-fidelity models of compressed video traffic for use in Monte-Carlo simulations of broadband communications networks but does not allow analytical tractability.

## 2.1.2 Voice Sources

Voice is transmitted over the packet network by digitizing and coding it using pulse code modulation (PCM) technique and then packetising it for transport. This comes under constant bit rate (CBR) service. A voice source alternates between talk spurs and silent periods. If the coding process uses silence suppression i.e. no packet is generated during the silent period or uses some compression scheme then the traffic generated will be variable bit rate (VBR) type. Some of the traffic models used for modeling packetised voice capturing the talk spurs and silent periods are Interrupted Poisson Process (IPP), On-Off process and 2-state MMPP [GRU82] [GRU83] [BRA91] [DAI91] [YIN91] [SRI86].

In On-Off process packets are generated at constant rate in the On-state (talk spurt duration) and no packets are generated in the Off-state (silent period). Both On and Off periods are assumed to be exponentially distributed. This model has been validated in [BRA91].

In Interrupted Poisson Process (IPP), the traffic switches between active period and silent period. As in the case of On-Off process the active period and silent period are assumed to be exponential distributed. In the active period packets are generated with packet interarrival times that are exponentially distributed. MMPP has been used to model the superposition of multiple voice sources [HEF86]. IPP may also be considered as a special case of MMPP.

### 2.1.3 Data Sources

Traditionally the interarrival time of data traffic in packet network has been modeled by exponential distribution. This model fails to account for the burstiness that exists in file transfer and web traffic [PAX95]. Based on the bimodal type message length distribution of traffic generated by higher protocol layers [GUS90], some of the models proposed for modeling data traffic in packet networks are Switched Poisson Process [HEF80] [KUC73], the Geometrically Modulated Deterministic Process (GMDP), and Switched Batch Bernoulli Process (SBBP) [HAS91]. Recent studies involving data traffic on Internet backbone [PAX95] as well as on local area network [LEL94] have shown persistence of traffic correlations and their presence at multiple time scales thus motivating the researchers to come up with self-similar nature of the traffic. Network traffic has similar statistical properties at a range of time scales: milliseconds, seconds, minutes, hours and even days and weeks. This characteristic is referred to as self-similarity. The heavy tailed, sub-exponential nature of the marginal distribution of the traffic prompted the usage of distributions like Pareto [LEL94] [PAX95] for modeling the data traffic on Internet. A probability distribution  $f(t)$  is heavy-tailed if  $F^C(t) \approx t^n g(t)$  for  $t \rightarrow \infty$  and  $n > 0$  with  $g(t)$  being a slowly varying function.

### 2.1.4 Superposed Traffic

There are two approaches for modeling the superposed traffic in packet networks like ATM. One is the statistics superposition where the statistics of individual processes is used to evaluate the statistics of the superposed traffic. This approach does not require the modeling of individual traffic stream, its computation is easy and the state space does not grow with the number of sources superposed. Only disadvantage of this method is that it may result in poor accuracy. In the model superposition approach, model for each individual process is determined. The model of superposed traffic is determined by superposing the model of each stream. This approach is applicable only when the individual traffics are modeled using models such as MMPPs, phase-type Markov renewals, Markovian arrival processes etc. For example the superposition of MMPPs is also an MMPP.

In [TAQ97] it has been shown that the superposition of many On/Off sources with strictly alternating on and off periods and whose on and off periods have high

variability or high variance, can produce aggregate network traffic that exhibits the self-similar or long range dependence effect.

The model of superposed traffic that takes into account the long-range dependence in the auto correlation in the traffic stream, is seen to be a better model. Though the presence of LRD phenomenon cannot be ignored, it has been argued that for networks with finite buffers it is sufficient to incorporate correlations upto some finite lag that is proportional to the buffer size [GRO90] [RYU96]. Hence the choice of Markov Models for modeling superposed traffic may be acceptable.

| Traffic Parameters | CBR | rt-VBR | nrt-VBR | UBR | ABR | GFR |
|--------------------|-----|--------|---------|-----|-----|-----|
| PCR                |     | Yes    |         | yes | yes | yes |
| SCR, MBS           | N/A |        | Yes     |     | N/A |     |
| MCR                |     |        | N/A     |     | yes | N/A |
| MCR, MBS, MFS      |     |        |         | N/A |     | yes |

CBR: Constant Bit Rate, rt-VBR: Real Time Variable Bit Rate, nrt-VBR: non-Real Time Variable Bit Rate, UBR: Unspecified Bit Rate, ABR: Available Bit Rate, GFR: Guaranteed Frame Rate

**Table 2.1 ATM Service Category Attributes**

ATM forum specifications on traffic management [ATM99] define traffic parameters to describe traffic characteristics of a source. A source traffic description constitutes of a set of traffic parameters of an ATM source. These parameters are provided by the user application when establishing a connection for the service. The choice of source traffic descriptor varies with the service category. Some of the traffic parameters defined in ATM specifications are Peak Cell Rate (PCR), Sustainable Cell Rate (SCR), Maximum Burst Size (MBS), Minimum Cell Rate (MCR) and Maximum Frame Size (MFS). List of traffic parameters required for each traffic category is given in Table 2.1.

In this work On-Off fluid traffic source model has been used. This has been done because of the tractability of this model and because it can capture most of the burstiness properties of broadband applications [XIO94]. Superposition of homogeneous On-Off fluid sources result in Markov Modulated Rate Process. Superposition of two Markov Modulated Rate Processes is also a Markov Modulated Rate Process. Thus allowing for tractable analysis of large systems.

## 2.2 Multiplexer Performance Analysis

Traditionally a network of queues has been analyzed by taking each queue independently. This is made possible because of Jackson's Theorem [JAC57]. The Jackson's network of queues model greatly simplifies the performance analysis of telecom networks with Poisson traffic arrivals and exponential service times. It reduces the analysis of the network into the analysis of individual communication links, each of which may be modeled as a M/M/m queue. This is possible due to the underlying Markovian nature of the traffic arrival process and service time distribution.

A result similar to Jackson's Theorem has been proved in [CHA98] using self-similar arrivals and deterministic service times. The following underlying properties of self-similar processes have been used for proving this result.

- The sum of independent self-similar processes is self-similar
- The random splitting of self-similar processes results in self-similar processes
- The output process of a G/D/1 queue with self similar arrival is self-similar

Node decomposition is another assumption that simplifies the analysis of networks of queues. This assumption means that the traffic characteristic does not change at the intermediate node in the network. By applying the nodal decomposition approach, it becomes tractable to analyze the performance of network as a whole. Besides simplifying the analysis, nodal-decomposition is also desirable for the robustness of a network [WOO90]. This assumption has been extensively used in performance analysis of traffic management schemes in intermediate nodes of the network. As the traffic passes through the intermediate nodes in the network, the burstiness in the traffic reduces because of the smoothening effect of statistical multiplexer [SAI91A] [OHB91] and superposition of a large number of bursty sources [SAI91A]. The inaccuracies arising due to the assumption of Node Decomposition method has been studied in [LAU93] [FRI93] [SAI91A].

In [OHB91] the traffic characteristic of tagged cells of a bursty traffic stream has been studied as they pass through the tandem queues. The authors have observed that as the load increases on the network the traffic characteristic of the stream under observation gets smoothed. The smoothening of traffic along the path results in different waiting time distribution, inter-cell departure time distribution and the coefficients of variations at successive nodes.

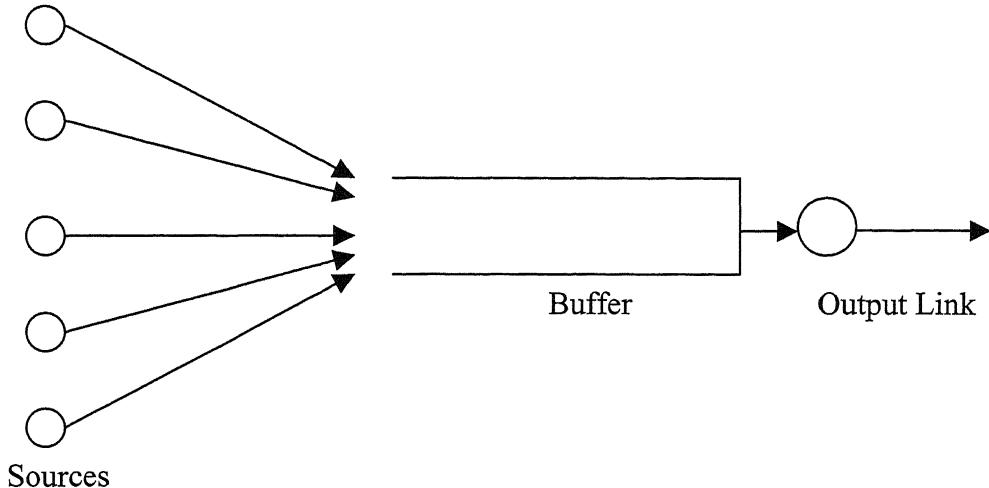
The effect of aggregation of traffic has also been observed in [SAI91A]. The burstiness of cell arrival process of multiplexed VBR voice sources decreases as the number of multiplexed source increases. The same was observed for multiplexed VBR video traffic.

The validity of nodal decomposition has been extensively studied in [LAU93] using simulation. The authors have considered both homogeneous and heterogeneous traffic environments. The individual sources are modeled as various 2-state/multiple state Markov-modulated processes. The nodal decomposition has been validated through studies on individual source departure characteristics and inter-source cross-correlation at the output of a network node. The traffic characteristics of the departure process were compared with those of the arrival process. It was observed that the output process of each individual source closely resembles to its original input process. This is particularly true if the link-speed is high enough so that no individual source can dominate the traffic on each link. The conditions under which the nodal decomposition is valid are:

- a) the peak rate of each source should not exceed 5% of the total link capacity,
- b) not more than 10% of the departing sources should go to the same immediate downstream link for inter source cross-correlation to have negligible impact on queuing performance of the downstream node.

These can be considered sufficient conditions for the validity of the nodal-decomposition assumption [LAU93].

The effect of multiplexing and switching on traffic characteristics and performance has also been studied in [FRI93]. It was observed that traffic arriving at a switching node differed from the traffic arriving at a user node. This has been attributed to the inherent smoothening that takes place when bursty cell streams are queued and then released at a given rate to the network. Some amount of smoothening was also observed as cells passed through the switching nodes. The authors have used the IPP and MMPP traffic models in their simulation studies. They have shown that the traffic descriptor peak bit rate, mean bit rate and mean burst length, provided by the ATM Forum Specification [ATM99], are not sufficient for the performance analysis of the multiplexer. This has also been shown in [RAT91]. The observations of the authors [FRI93] on smoothening effect of the multiplexer may be due to the fact that the traffic conditions used for simulation by the authors violated the conditions laid down in [LAU93] for validity of nodal decomposition.



**Figure 2.1 ATM Multiplexer**

The performance analysis of a queuing network generally deals with analysis of loss and delay performance at each node within the network, assuming the validity of nodal decomposition approach. The cells arriving at the ATM multiplexer (Figure 2.1) are queued in a buffer and served on the basis of some scheduling algorithm (e.g. FCFS). The output link is the server for the multiplexer. For any loss performance analysis of a queue, one needs to define the arrival process, the service time distribution, number of buffers in the queue and the number of servers. This is usually a single server queue.

The basic function of call admission control policy in ATM Networks is to decide whether to accept or to reject a new request for connections. This decision is either directly or indirectly based on the estimate of the cell loss probability for each class of existing calls, given that the new connection has been accepted. The analysis of the loss probability becomes a central issue in the application of queuing theory to ATM Networks and has attracted considerable attention from researchers. A number of researchers have made contributions to the techniques for performance evaluation of a statistical multiplexer with correlated arrivals [ANI82] [HEF86] [BAI91] [CHE93] [SAI91A] [HUE98] [TRA88] [BLO92] [DIN90] [HER93] [YAN79] [GRA85] [BAI93] [KOS84] [STE91] [NAR98] [NOR91] [NEU81] [CHO] [SAI90] [TUL98] [ROB91] [NAG91]. Some of the queuing models proposed in the literature for modeling packet multiplexer in general and ATM multiplexer in particular are discussed here along with the methodology used for analytical treatment of the queuing model. Only a few input processes allow for the exact analytical treatment of a finite queue, which is fed by

those arrival processes. No simple closed form solutions are available for the correlated arrival process used in the analysis of the ATM statistical multiplexer.

Since it is difficult to solve analytically the equations that determine the characteristics of the queuing systems of the type  $GI/G/1$  and similar systems, various inequalities between characteristics of these systems and the corresponding characteristics of simpler systems have become important. These are based on stochastic ordering and have been used by researchers to obtain approximate but accurate solutions for the complex queuing models.

The loss probability in the finite queue is often approximated by the probability that the corresponding infinite queue, with the same arrival and service processes, exceeds the finite queue length. This approximation allows for the analytical treatment of a wide range of arrival processes since infinite queues are often amenable to exact analytical solutions while finite queues are not. The accuracy of this approximation has been investigated in [HUE98]. The authors have observed that the approximation is conservative for the arrival processes with short-range dependence as long as the load and the loss probabilities are low. For higher loads and loss probabilities, the approximation overestimates the exact loss probability by several orders of magnitude and larger queue sizes lead to large inaccuracies in approximation. For the arrival processes with long-range dependence, the approximation results in several orders of magnitude difference for almost all load levels.

Neuts [NEU81] has derived complex closed formulae for phase type arrival and service time distributions using  $M/PH/1/K$  and  $PH/M/1/K$  queues, but these lend themselves to little more than numerical computation especially for large dimensionalities of the state space of the PH distribution. Even the well known  $M/G/1/K$  and  $GI/M/1/K$  queues can be analyzed only numerically, for example by using the Embedded Markov Chain approach. The problem of determining the explicit accurate approximation of the loss probability for the  $M/G/1/K$  and the  $GI/M/1/K$  queues has been addressed in [BAI93]. The most interesting properties of these approximate formulae are that

- a) they are asymptotically correct i.e. the ratio of the exact value of the loss probability to the approximate value tends to 1 as the buffer size increases [BAI92],
- b) they reduce to the exact expression of the loss probability in the case of the  $M/M/1/L$  queues,

- c) their coefficients can be easily computed from the main queue parameters and
- d) they can be inverted allowing a straight forward buffer dimensioning to be done.

There are mainly two approaches to evaluate the performance of an ATM multiplexer, namely a fluid approximation and a matrix analytical approach. Both the approaches have their advantages and limitations. The matrix analytical approach takes advantage of the special structural properties that are used to construct workable algorithms even when the problem is inherently of high dimension. The mathematical structure of the models is particularly useful in the discussion of steady-state features. The various works discussed here bring about the limitations and the advantages of the techniques based on the two approaches.

Discrete-time finite buffer queues with correlated inputs have been used [BLO92] for calculating the queue length density/loss probability using the DBMAP/D/1/K queue. Both Discrete-Batch Markovian Arrival Process (DBMAP) and Discrete Markovian Arrival Process (DMAP) are derived from the continuous time analogue BMAP and MAP [LUC91]. The structure of transition matrix  $Q$  shows that the queuing system is a finite Markov Chain of  $M/G/1$  type. The method based on result of [GRA85] has been extended to block matrices for analyzing the  $DBMAP/D/1/K$  queue. This allows computationally tractable solutions for various performance measures such as the buffer occupancy distribution, cell loss probability and conditional cell loss probability. This approach leads to an accurate but conservative approximation for a large range of system parameters.

An important property of MAPs and BMAPs is that superposition of independent processes of these types are again processes of the same type. This property is exploited in [CHO] to study the effect of statistical multiplexing of a large number of bursty sources. The Markov Arrival Process includes as special cases both the phase-type renewal process [NEU81] and the Markov Modulated Poisson Process [HEF86]. The transient results for  $BMAP/G/1$  queue have been obtained in [LUC] following the matrix generalization of transient results for the  $M/G/1$  queue in [TAK62]. The transient behavior of  $M/G/1$  type queue has been used to calculate the time-independent probability distributions by numerically inverting the two-dimensional transforms. For this purpose the two-dimensional transform inversion algorithm, proposed in [CHO1] has been used. These algorithms are based on the

Fourier- Series method [ABA92] and are enhancement and generalization of the Euler and Lattice Poisson algorithms. The busy period transforms are obtained by iterating the characteristic functional equations. In [TUL98] the explicit expression for the transform of the queue length at the  $n^{th}$  departure has been obtained, assuming a departure at  $t=0$ , and the workload at the  $n^{th}$  arrival, keeping track of the appropriate phase change. The departure process is characterized by the double transform of the probability that the  $n^{th}$  departure occurs at time less than or equal to time  $x$ . The result is found to be similar to that derived in [SAI90].

The superposed sources and the statistical multiplexer have been modeled by means of a two-state MMPP/D/1/K queue in [BAI91]. The superposition of high-speed On-Off sources has been modeled using 2-state MMPP. The equilibrium analysis available in [BLO89] has been used along with a fast algorithm [BAI91] for queue analysis whose computational complexity does not depend on the buffer size  $K$ . In [BOU91] the  $MMPP/D/1$  model has been used for obtaining the loss probability. The upper block Hessenberg structure of the transition matrix has been exploited for obtaining the stationary probabilities. MBH algorithm has been used for efficient computation of loss probability using matrix-geometric approach. In [NAG91] the  $MMPP/D/1/K$  queue has been analyzed using the technique of uniformization in a fashion similar to that in [GRA82] and [LUC85] in the analysis of phase type queues. MMPP predictions are found to be very close to the results obtained from simulation.

$MMPP/G/1$  queue has been used in [HEF86] to approximate the performance of a statistical multiplexer with inputs consisting of the superposition of voice streams and data streams. As in the case of  $M/G/1$  queue analysis, the stationary distribution of the Markov chain is obtained from the first two moments of the virtual waiting time. Powerful matrix-analytic techniques have been used to obtain the tails of voice packet delay distribution. The analytic approach to the stationary solution of  $N/G/1/K$  queue has a key ingredient which requires the solution of probability transition matrices of the imbedded Markov process,  $A$  and  $A_n$ . In general, the iterative procedures based on the randomization techniques as suggested in [GRA82] and [BLO89] can numerically solve these matrices. It has been shown in [CHE93] that in the case of  $MMPP/G/1/K$  queue there exists explicit expressions for computing the matrices  $A$  and  $A_n$ . This is achieved by using the matrix spectral decomposition, which only depends on the computation of eigenvalues and associated eigenvectors of the system. In case where

the MMPP is the superposition of homogeneous IPPs, the eigenvalues and eigenvectors can be given in closed form.

Quasi-Stationary approximation has been used in [CHE93] to obtain the approximate cell loss probability for an ATM statistical multiplexer with MMPP arrival process. In quasi-stationary approximation, use is made of the fact that changes in the number of active sources occur at a moderate rate. It is assumed that the time for the queue to reach the steady state after a change is short compared to the time to the next change in the number of active source. The queue follows this slow variation in the number of active traffic sources, such that during the successive changes the cell loss probability appears stationary. Thus the underlying queuing system becomes the  $M/G/1/K$  queue. The appropriateness of this model has been examined through the transient analysis of the underlying process.

Output queue at an ATM node has been approximated by a generic queuing model  $\sum G_i/D_i/I_i/K_i$  for analysis [SAI91A]. For analytical ease, the arrival process  $G_i$  is modeled as MMPP, and the service time distribution  $D$  by the  $L^{\text{th}}$  order Erlang distribution ( $E_L$ ). The superposition process itself has been modeled by an MMPP and has been obtained from the infinitesimal generator  $M_i$  and arrival rate matrix  $\Lambda_i$  of individual processes. If  $X(t)$  is the number of cells in the system at  $t$ ,  $J(t)$  is the cell arrival phase and  $L(t)$  is the service phase at time  $t$  then the triplet  $[X(t), J(t), L(t)]$  in an  $MMPP/E_L/I/K$  queue forms a Markov process with infinitesimal Generator  $M$ . The stationary state probability  $p(i,j,k)$  at an arbitrary point is obtained by numerically solving the following equations

$$p \cdot M = 0 \quad M \cdot I = 0 \quad p \cdot I = 1 \quad (2.2)$$

The algorithm used is such that the processing time is independent of phase size  $L$  of the service time distribution. The stationary state probability is used for obtaining the performance measures like cell loss probability, probability of delay exceeding maximum delay etc.

$MMPP/PH/1/K$  queuing model has been used in [YAM93] for obtaining the loss period and non-lossy periods. The Laplace transforms of the probability distribution function of these times have been obtained by recursive formulae. These distributions have been obtained via the Laguerre transform method.

The discrete-time  $SSMP/G/1$  queue has been used in [DIN90]. This is used to obtain an estimate for the case of a finite buffer queue, when the probability of loss is

sufficiently small. The exact analysis of the discrete-time  $SMP/D/1/s$  queue [HER93] has been used to model the behavior of ATM statistical multiplexer and has been shown to be useful in dimensioning of real systems. The method of unfinished work at arrivals, described in [TRA88] for  $G^{[X]}/D/1/s$  queue, has been used to obtain the exact analysis of the discrete-time SMP/D/1/s queue yielding easy expression for both the queue length density at arrivals and the waiting time density for the “arrival first” and “departure first” policies. The input process is an arbitrary SMP (Semi Markov Process), the state of which corresponds to simple arrivals. The result developed can also be used for SSMP and DMAP processes.

Renewal Approximation is used in [NAG91] to model the ATM statistical multiplexer. The superimposed voice arrival process has been modeled as a renewal process. The packet loss has been obtained for the  $GI/D/1/K$  system by obtaining the first and second moments for the infinite buffer queue using approximate formulae from [WHI83]. A discrete or continuous distribution is then chosen to match the computed moments to approximate the probability distribution for queue occupancy in the infinite buffer. The probability distribution for finite buffer is obtained from that for infinite buffer. A mixture of geometric distributions (analogous to the hyper-exponential distribution or a mixture of exponential distribution in the continuous random variable case) has been used. The Renewal approximation is the simplest and most versatile of all models. However due to its limited accuracy it can most probably be employed only in heavy traffic situation where it is relatively more accurate.

$MMDP/D/1/K$  queue has been used in [YAN79] due to its computational simplicity and accuracy. An MMDP is a deterministic process whose arrival rate is controlled by a finite continuous time Markov process. The  $MMDP/D/1/K$  queuing system has been analyzed using the method proposed in [YAN92].

The ATM multiplexer has been approximately modeled by  $N*D/D/I$  queue in [ROB91]. A multiplexer of capacity one cell per time unit handling  $N$  sources has been considered. Each source transmits one cell every  $D$  time units.  $D$  is the ratio of the multiplexer capacity to the arrival stream bit rate. The  $N$  cell arrival epochs in an interval  $[t-D, t)$  are independently and uniformly distributed in this interval.

The discrete-time  $AR(1)/D/I$  queue has been analyzed in [HUE98], the solution to this model is based on an iteration approach. The exact solution for the infinite FBM (Fractional Brownian Motion) driven queue has been proposed in [NAR98]. For

simulation purpose, FBM process arrivals were generated using the Random Midpoint Displacement Algorithm [LAU95].

In Fluid Flow model the discrete nature of packet arrival is approximated by a continuous flow. The small size of packets/cells in ATM permits the use of fluid flow model for analyzing the queuing behavior at an ATM multiplexer. Markov modulated Rate Process or fluid flow model has been extensively analyzed. The analytical techniques can be categorized into exact analysis, analysis using simplifying approximations and asymptotic approximation. The disadvantage of the fluid flow approximation is that cell level behavior is totally lost. It accurately captures the correlated properties of the superposition but fails to account for the stochastics in the flows [NAG91]. This is the reason that the fluid flow model fails for small buffers where the stochastics are more significant than the correlations. It also exhibits numerical instability in case of large source population and for buffer sizes [KOS84]. The reason for this being that the set of linear differential equations governing the behavior of the equilibrium probabilities has unstable eigenvalues. The inevitable errors, no matter how small, incurred during numerical integration are liable to excite the unstable modes and lead to solutions that blow up (similar to the case when the Laplace transforms of the equilibrium probabilities are available and are to be numerically inverted). In [ANI82] this problem has been countered by a priori segregation of the stable modes from the unstable modes. This requires the complete knowledge of eigenvalues and eigenvectors, information that is generally costly to obtain. The effectiveness of the solution proposed in [ANI82] depends on the efficiency of the computation of the eigenvalues and coefficient vectors. This has been achieved by obtaining all the eigenvalues by solving only a set of quadratic equations. A noteworthy feature of the approach proposed in [ANI82] is that it manages to avoid requiring the numerical solution of matrix equations. In [BAI91] it has been shown that increasing the buffer size leads to an increase in the number of correlation components summing up and consequently the cell loss probability does not significantly decrease. Hence increase in the buffer size is practically ineffective beyond a certain size corresponding to the knee of loss probability versus buffer size curve. Such an effect is more and more pronounced under increasing mean offered load. It has also been shown [BAI91] that  $M/D/1$  model gives a good estimate in the small buffer region. For small buffer sizes, the bimodal distribution follows. The queue is alternatively almost full or almost empty. This phenomenon is all the more remarkable as the mean offered load

increases. Hence limiting the maximum allowable load to values such as 0.3 or 0.4 should be considered.

A broad class of buffered system has been examined in [STE91] by modeling the system as continuous flow process modulated by an underlying reversible Markov Process. It has been shown that if the modulating process is separable, considerable reduction in computational complexity is possible using a decomposition method initially proposed in [KOS84]. This avoids problem of state explosion and numerical accuracy in very large systems. Further reduction in computational complexity was shown to be possible using simple bounds on coefficients in the expression for the buffer occupancy distribution. It has also been shown that certain computational problem associated with matrix-geometric methods can be avoided using decomposition. The decomposition technique leads to a solution of the equilibrium equation expressed as a sum of terms in Kronecker product from. A key consequence of decomposition is that the computational complexity of the problem is vastly reduced for large systems.

In [NOR91] it has been shown that fluid model gives an exact expression of one component of real queue. The load fluctuation of the cell arrival rate about the fluid average has been considered. The fluid approximation for the superposition queue can be more satisfactorily viewed as a way to calculate the “burst component” of the real queue, to which a further “cell component” must be added, depending on local fluctuations in the arrival rate about the fluid average. The component obtained from the fluid flow model with correlated input accounts for the long-term positive correlation in the arrival rate process. The negative correlation between cell inter arrival time is taken care of in second component. The cell component has been approximated by the  $\sum D/D/1$  queue taking the burst component to be zero. The queuing behavior for the correlation arising in the cell arrival process when variable bit rate sources are multiplexed in an ATM network has been analyzed using the general result for the  $G/G/1$  queue due to Benes [BEN63]. The cell component is seen to be contributing a small positive bias, which approximately equals the mean of a  $\sum D/D/1$  queue with a load equal to 1. It has been observed that the first moment of delay distribution may depend significantly on the cell component while the low percentile necessary for the buffer dimensioning are given essentially by the burst component alone.

Non-parametric approach has been used to obtain the upper bounds of the cell-loss probability [SAI93]. This is useful under the condition, where only the traffic parameters are given and no arrival process is defined. The evaluation method that does not assume a parametric cell arrival process is called the non-parametric approach. A performance measure like loss probability cannot be obtained using this approach. Only upper bound for the loss probability can be computed using non-parametric approach.

The fluid flow approach has been used in this thesis. The fluid flow approach results in more tractable model than the actual queuing model and does not suffer from what is termed as curse of dimensionality.

## 2.3 Traffic Management

The aim of traffic management is to control network congestion, efficiently utilize network resources and deliver the negotiated quality of service to users. The traffic management in ATM Networks is based on the simple concept of allocating resources within the network for each connection so as to provide appropriately differentiated Quality of Service (QoS) [Section 1.2.4]. This section covers the various methods proposed for implementing the Call Admission Control and Usage Parameter Control.

### 2.3.1 Call Admission Control

One of the major challenges in ATM based broadband network is to guarantee the promised QoS for all admitted users while maximizing the resource utilization through dynamic resource sharing, i.e. statistical multiplexing of traffic streams [LIE95]. Call admission control, bandwidth allocation and buffer management play a key role in achieving the promised QoS for every connection or every class of connections.

The Connection Admission Control (CAC) function is defined in Section 1.2.4. The function uses the information contained in the traffic description provided by the user at connection setup time to determine whether the request for a new connection should be accepted or rejected. The Call Admission is based on the availability of resources (bandwidth and buffers) to ensure that the QoS requested by the new connection can be guaranteed without degrading the QoS of the existing connections [ATM99]. The CAC has to operate in real time hence the computational burden must be consistent with the available processing capacity on the switch. Given the statistical

nature of the offered traffic, the vague description of the traffic characteristics provided by the users [Table 2.1], the stringent requirements on QoS, and the need to maintain a reasonable level of utilization, it is a significant challenge to develop an efficient and robust CAC.

The literature has a large number of Call Admission Control strategies proposed by various researchers working in this area. Most of the Call Admission strategies are based on mathematical computation of the approximate loss probability and performing CAC according to these estimations. CAC algorithms can be broadly classified into

- a) Equivalent bandwidth approximation [ELW93] [GUE91] [KES93] [CHA95]
- b) those based on engineering of loss probabilities [BAI91] [CHE93] [CHO96] [ELW95A] [LAB92] [SHI97] [RAM97][MUD95]
- c) Upper bounds based on average and peak rate combinatorics [LEE96] [FER90] [CHO98]
- d) those based on large deviation theory [BOT95] [GLY94]
- e) mechanisms based on Artificial Intelligence techniques [ATL00]
- f) mechanisms based on measurements [GIB95] [SAI91][TZE][ZHU96]
- g) Fast resource reservation and Time window mechanisms [TUR92] [DZI96].

Various effective bandwidth methods have been proposed in the literature [ELW93] [GUE91] [KES93] [CHO96] [ELW95A] [CHA95]. Effective bandwidth, which lies between average and peak rate, is computed for each flow. The stochastic properties of the flow have been taken into account for obtaining effective bandwidth of each flow such that the required loss probability for the flow is satisfied. Most of the techniques use the tail probability  $P[Queue\text{-}size > Buffer]$  for obtaining the effective bandwidth of a flow. Some of the approaches used for computing effective bandwidth are based on eigenvalue decomposition of Markovian flows [ELW93], large deviation theory [KES93] and the theory of envelope processes [CHA82]. The additive effective bandwidth of a flow is independent of the properties of all other flows as well as the number of sources and the link capacity  $C$ . In most of the cases the loss probability  $P_{LOSS}$  has been approximated by

$$P_{LOSS} \sim e^{-bX_B} \quad (2.3)$$

to obtain the additive effective bandwidth, where  $X_B$  is the buffer size and  $b$  is a function of effective bandwidth. The call admission control procedure uses this additive effective bandwidth for accepting or rejecting a call.

$$\sum_{i=1}^N C_i < C \quad (2.4)$$

Here  $N$  is the number of multiplexed sources,  $C$  is the capacity of output link and  $C_i$  is the effective bandwidth of the  $i^{th}$  flow. The  $e^{-bB}$  approximation for the loss curve is found to be highly conservative in nature. This method does not take advantage of the statistical multiplexing of large number of sources and hence again results in under utilization of link capacity.

A common approach to deal with this problem is to assume that the buffer content process  $X(t)$  has an exponential complementary queue-length distribution.

$$P[X(t) > x] \approx ae^{-bx} \quad (2.5)$$

The constants  $a$  and  $b$  may be explicitly represented in terms of the parameters and statistics of the input process through analysis and approximation [CHO96] [ELW95A] [MUD95]. A more general approach for the estimation of  $a$  and  $b$  has been used by applying large deviation theory for estimating  $a$  and heavy traffic expansion for computing  $b$  [SHI97].

In [CHO96] [MUD95] [ELW95A] the efforts have been made to improve the loss curve approximation by computing the approximate asymptotic constant  $a$  in Equation 2.5. In [MUD95] the asymptotic constant has been obtained by using the exact computation for Markov modulated fluid sources based on [ANI82] and also by Gaussian approximation for the overload probability. This approach is described in detail in Chapter 6 of this thesis. In [ELW95A] the asymptotic constant has been estimated by the loss probability in a bufferless multiplexer as estimated by Chernoff's theorem. This approach of computing the effective bandwidth of the combined traffic works well for most Markovian sources but has been found inaccurate for sources with long range dependence. The tail probabilities of multiplexer fed with self-similar traffic [LEL94] are of sub-exponential type.

In [RAM97] a model based multiclass connection admission control scheme has been developed. CAC algorithms compute the amount of bandwidth needed to support the specified QoS for each class of traffic. For buffered systems, the aggregate traffic

process is modeled by a versatile diffusion process, and the results from the earlier work in diffusion approximations have been applied so that the auto-correlation of traffic is also taken into account.

In [LEE96] [FER90] average and peak rate combinatorics have been used to obtain approximate loss probability for a bufferless multiplexer loaded with On-Off sources. An efficient algorithm has been developed in [LEE96] for computation of the aggregate arrival rate distribution and subsequently the loss probability. In [FER90] the worst case average rate for any time interval has been used to obtain the probability of delay-bound violation. Assuming a bufferless model results in substantial under utilization of link capacity if the multiplexer does contain a buffer [KNI99].

Maximum Variance Approach has been used to obtain Call Admission Control Algorithms [CHO98] [KIM]. In this approach a variable  $Q_t$  is defined as

$$Q_t = \sum_i A_i[s-t, s] - Ct \quad (2.6)$$

where  $C$  is the output link capacity and  $A_i(t)$  is the instantaneous arrival from  $i^{th}$  source at time  $t$ . The tail probability for the buffer occupancy distribution is given by

$$P(X > X_B) = P(\sup_{t \geq 0} Q_t > X_B) \quad (2.7)$$

The accurate bounds have been derived using the above definitions and assuming  $Q_t$  to be Gaussian [CHO98]. The empirical studies in [CHO98] have shown that accurate results are obtained when the arrival process can be effectively modeled as a Gaussian process. Maximum Variance Asymptotic upper bound has been obtained based on the normalized maximum variance  $\sigma_B^2$ .

$$\sigma_B^2 = \max_t \frac{\text{var}\{Q_t\}}{(B - E(QA_t))^2} \quad (2.8)$$

The MVA approach has been extended in [KIM] to estimate loss probability by normalizing the MVA upper bound by the exact probability of loss in a bufferless system.

Large deviation theory has been used in [GLY94] to show that in large class of stochastic processes the tail probability satisfies the following equation

$$\log P(X > X_B) \sim -\delta X_B \quad (2.9)$$

However, for sources whose flow shows long range dependence (e.g. self-similar traffic) the tail probability has been defined by extending the result from large deviation theory to more general large deviation techniques [DUF95] to obtain

$$\log P(X > X_B) \sim g(X_B) \quad (2.10)$$

The effective bandwidth obtained using the large deviation theory is not additive hence the resources required for the source depend on all other sources. These can also address the cases of non-Gaussian traffic but are computationally expensive in the calculation of supremums [KNI99].

In all the Call Admission strategies discussed so far, the CAC requires each source to characterize the offered traffic a priori in terms of the parameters of a deterministic or stochastic model. The connection admission controller makes a decision based on the traffic parameters of existing connections and the new connection, which are usually declared by the users at connection setup time. Admission controls of this kind are open-loop model-based CAC [ELW95] [GUE91]. Such methods have some associated drawbacks. Often it is difficult for the users to precisely characterize their traffic streams since

- a) the statistical nature of traffic sources may not be completely captured by the analytical model,
- b) in order to conform to the standards, the choice of parameters may be inadequate to completely characterize the traffic source.

Another drawback of these schemes is that each of them is based on specific traffic models and that they can be applied only to sources that comply with these models. To make matters worse, traffic analysis usually involves complex and computationally expensive procedures and therefore is not practical in real time. Given the limitations of traffic source descriptors, in order to provide guarantees on QoS, the CAC has to allocate bandwidth based on a worst-case scenario that the declared traffic descriptors may represent. This can result in more bandwidth being allocated than needed, leading to under-utilization of network resources. Inside the network, due to queuing, traffic superposition and thinning, traffic streams may no longer conform to UPC parameters declared originally and consequently may require more bandwidth than what was allocated by the CAC. Here it is assumed that the policing (UPC mechanism) is performed only at the ingress of the network. However the most significant drawback is that many of them overestimate the effective bandwidth or the

expected cell loss probability, which results in under utilization of the network resources. Moreover their effectiveness depends considerably not only on the utilization of the network resources and the QoS contracts but also on the statistical multiplexing that is achieved [GUE91]

On the other hand, the mechanisms based on Artificial Intelligence techniques, or a combination of them with fuzzy logic, do not depend on specific traffic models. The Artificial Neural Networks (ANNs) are trained off-line according to specific traffic patterns and QoS requirements. However their convergence to the correct decision (acceptance or rejection of the call request) during their on-line operation depends heavily on whether the real traffic characteristics resemble the ones that the ANN has been trained for. As the scenarios that the ANN should be trained for increase, so does its convergence time, its size and the probability of converging on a wrong answer. The cost of their implementation considerably increases and it is possible to cause congestion in the network due to a wrong decision.

A real time dynamic CAC algorithm has been proposed in [RAM97]. The CAC algorithm explicitly computes the effective bandwidth required to support each class of connection based on on-line observations of aggregate traffic statistics as well as the declared parameters. Gaussian and diffusion approximations have been used to characterize the aggregate traffic stream, and use fuzzy control strategy to combine model and measurement results to derive simple closed-form formulas to estimate the effective bandwidth in real time.

In particular, the methodology of fuzzy logic controller appears to be very useful when the processes are too complex for analysis by conventional quantitative techniques, or when the available information cannot be interpreted correctly and with certainty.

The measurement based admission control does not make any assumption about the offered traffic. Instead, the network monitors and measures the incoming traffic statistics, and makes decisions to admit or reject based on the measured statistics. The measured number of cells arriving during a fixed interval is used in [SAI91] to derive a robust but loose upper bound for cell loss ratio.

A decision-theoretic approach for call admission control to explicitly incorporate call-level dynamics into their model has been used in [GIB95]. Call acceptance decisions are based on whether the current measured load is less than a pre-

computed threshold. Bayesian decision theory provides framework for the choice of threshold.

A robust measurement based admission control with emphasis on the impact of estimation errors, measurement memory, dynamics and separation of time-scale has been studied in [TZE]. They quantify the impact on the effective bandwidth of large systems using Gaussian and heavy traffic approximation at the call level.

Kalman filtering has been used for an optimal estimation of the aggregate effective bandwidth  $C$ . The effective bandwidth is expressed in the form of

$$C = \alpha \bar{R} + \beta \sigma^2 + R_e \quad (2.11)$$

where  $\alpha$  and  $\beta$  are preset parameters.  $\bar{R}$  is estimated mean of the aggregate traffic rate,  $\sigma^2$  is estimated variance of the aggregate traffic rate and  $R_e$  accounts for the estimation errors under Gaussian assumption. Since the required QoS is in general stringent, a straight forward, on-line monitoring of such rare events will take too long to be practical. A common approach to deal with this problem is to assume that the buffer content process  $X(t)$  has an exponential complementary queue-length distribution.

$$P[X(t) > x] \approx ae^{-bx} \quad (2.12)$$

where parameters  $a$  and  $b$  are to be estimated based on on-line measurements.

In [ZHU96] the  $P[X(t) > x]$  has been measured for threshold  $x << X_B$  (buffer size), such that the  $P[X(t) > x]$  are in the range of  $10^{-2} - 10^{-3}$ . The autoregressive (AR) method has been used to estimate the parameters  $a$  and  $b$  from measurements. Then the above equation is used to obtain  $Prob[X(t) > X_B]$ .

While measurement based admission control schemes capture the on-line traffic dynamics more faithfully and achieve higher efficiency of bandwidth allocation, sole reliance on measured and estimated quantities for admission control may raise new issues and lead to uncertainties. Monitoring and measuring traffic statistics requires additional processing capacity in the network. It is an excessive processing burden. Second any measurement or estimation procedure has errors associated with it, e.g. feasibility of accurately measuring certain statistics and the inherent disparity between the estimation models and the measured statistics.

Most of the call admission policies are based on complete sharing of resources (bandwidth and buffers). In all these schemes, all the available link capacity is open to all classes of users. The admission controller, under the complete sharing policy,

determines if there is enough capacity to admit a newly arriving call and admits or rejects the call depending on whether or not there is enough capacity. This does not meet the call blocking probability of different call classes.

The complete partitioning strategies [MIT98] [BUR98], divide the entire capacity into as many partitions as there are classes of users, a new user that finds insufficient capacity in the partition for its class is dropped. The complete partitioning scheme can be tuned to control the relationship between the call blocking probabilities of various classes of users, but the resulting throughput may be very low.

A hybrid of the complete sharing and complete partitioning of bandwidth is the partial sharing policy. In the partial sharing policy, each call class has a dedicated portion of the bandwidth reserved for it, and all the classes compete for the remaining unreserved portion of the bandwidth [LAB92].

The ATM forum traffic management specification [ATM99] does not specify any Call Admission Control function. The implementations of these functions are network specific and depend on the strategy implemented by the manufacturer of ATM network element (switches, multiplexers, etc).

### 2.3.2 Usage Parameter Control

While the Call Admission Control functions in ATM networks accept/reject a call request and manage network resources accordingly, Usage Parameter Control (UPC) functions perform the function of policing the traffic on Virtual Connections (VCs) and Virtual Paths (VPs) [Section 1.2.4]. The ATM Forum Traffic Management Specifications [ATM99] define Usage Parameter Control as the set of actions taken by the network to monitor and enforce the traffic contract. Its main purpose is to protect network resources from the malicious as well as unintentional misbehavior, which can affect the QoS of other established connections. This protection is achieved by detecting violations of negotiated parameters and taking appropriate actions.

UPC is performed at the UNI (User Network Interface) and NNI (Network Network Interface) for each VC or VP. The policing functions include the checking of the validity of VPI and VCI for each VC (and VPI for each VP), count the number of arriving cells belonging to the connection setup, and impose a penalty on the connection if there is disconformity. UPC is performed for each traffic parameter in a source traffic descriptor. As of now, the ATM forum specifications only include peak cell rate as the parameter to be monitored for conformance to the traffic contract.

If UPC detects a violation, it can either discard cells or tag them for discard when the network is congested, or may allow them to pass [ATM99]. Tagging is done by making use of the cell loss priority bit in the cell header. When a cell with  $CLP=0$  is tagged, UPC sets the  $CLP$  of that cell to  $1$ . The tagged cell becomes indistinguishable from the cells that originally had  $CLP=1$ . The recommendations in [1] requires that UPC function should monitor both the streams (with  $CLP=0$  and  $CLP=1$ ) for a connection when monitoring for PCR. Cell Delay Variation (CDV) can make UPC inaccurate, mistakenly detecting a violation and discarding or tagging a cell conforming to the traffic descriptor. This is an important aspect in performance of UPC function.

The most famous algorithm for judging a violation is called the leaky bucket method or the virtual leaky bucket method [BAL90]. This method provides a pseudo buffer and whenever a user sends a cell, the queue in a pseudo-buffer is increased by one. The pseudo-server serves the queue and the service-time distribution is constant. As long as the queue does not overflow the finite pseudo-buffer, the cells pass the UPC without interference. If the pseudo-buffer overflows, the UPC judges that violation occurs and discards or tags the arriving cell. The service rate of the pseudo-buffer usually corresponds to the rate to be policed (PCR in this case). The pseudo-buffer assures that the UPC algorithm tolerates for traffic fluctuations caused by the CDV. Accordingly the pseudo-buffer size can be set. The performance of a leaky bucket degrades as the pseudo-buffer size increases [HSI93]. Various versions of leaky bucket algorithm have been proposed in literature [ONV94].

The other UPC algorithms are based on counting of the number of cells in a window such as jumping window method, the sliding window method, and the exponential window method. The jumping window method defines the windows that do not overlap. The occurrence of a violation is judged by comparing the number of cells in each window with a threshold value. The sliding window method defines windows overlapping with each other. The exponential window method defines a window in the same way as that in jumping window. It compares the number of cells in each window with a threshold value that is a function of the exponentially weighed sum of cells accepted in the preceding windows. The jumping window method is the simplest but its performance is poor.

It is generally agreed that the leaky bucket methods perform better than the window-based schemes. The leaky bucket and the exponentially weighed moving average have been found to be the most effective of all the UPC mechanisms proposed

in the literature [ONV94]. Generic Cell Rate Algorithm (GCRA) has been used to define conformance with respect to the traffic contract [ATM99] but the network may still use any UPC mechanism as long as it supports the QoS objectives of a compliant connection.

GCRA is a virtual scheduling algorithm or a continuous-state Leaky Bucket Algorithm. It is used to define the relationship between PCR and the CDVT, and the relationship between SCR and the BT (Burst Tolerance). Multiple instances of the GCRA may be applied to different cell streams of the same connection. Detailed flow chart of this algorithm is given in [ITU96] [ATM99].

## 2.4 Feedback based flow control

The congestion control schemes discussed in the previous section are mainly preventive in nature. Preventive methods of congestion control work by avoiding or reducing the possibility of congestion in the network through under utilization of resources. These schemes are conservative as these are based on the philosophy of bounding tail probabilities of delay distributions and minimizing packet losses due to overflows. Another characteristic of the preventive schemes is that the local problems are tackled locally. The traffic management functions like CAC, UPC, defined in ATM Forum specification are preventive congestion control methods.

Although preventive techniques reduce the buffer overflow probabilities, it is not possible to eliminate momentary periods of cell losses totally due to statistical nature of multiplexing in the network switches. Lost cells result in retransmissions, which in turn result in further losses thus turning the momentary buffer overflows to sustained periods of cell losses. Therefore, in addition to preventive schemes, reactive control mechanisms are necessary to monitor the congestion level in the network and take action based on that information. ATM Forum traffic management specifications define reactive control mechanism for ABR traffic in form of ABR flow control [ATM99]. There is no reactive control mechanism for traffic in other service categories.

In this section, some of the traditional reactive control mechanisms based on feedback from the congested node are discussed along with some of the new strategies proposed in the literature on high speed networks. Traditionally the packet networks were designed to support the data traffic that was not so sensitive to delay in the network as the real time traffic. Most of the reactive flow control schemes were

designed for low speed data networks hence the conventional reactive control mechanisms are not as effective in ATM networks as they are in low speed networks. In all these mechanisms, the basic philosophy was that the source generated traffic at the rate that was governed by the level of congestion in the network. There was no allocation of bandwidth at the setup time and the bandwidth was allocated dynamically and it kept changing as the congestion within the network changed. The ABR service category in ATM networks is also based on this philosophy that traffic generated by the source depends on the available resources in the network and bandwidth is allocated dynamically within an ATM network, hence the name Available Bit Rate Service.

The design and implementation of a generic reactive control mechanism covering the diverse traffic sources having different traffic characteristics and service requirements is still an open issue. Most of the effort in flow control mechanism for high-speed networks has been directed towards developing effective mechanism for reactive control for ABR flow [CHE96] [FEN96] [JAI96] [PRA98] [YIN94] [BEN98]. What is required here is to have a new look at the fundamental principles in congestion control for end systems and network elements in the context of a network that supports several services and QoS guarantees [GOY99].

The traditional flow control mechanisms are based on end node notification techniques like estimation by the end nodes, explicit backward congestion notification (EBCN) and explicit forward congestion notification (EFCN). It is this feedback information from the network or from the destination node, which is used by the source to increase or decrease its rate or window size (in window based mechanisms).

The ABR flow control uses the forward RM (Resource Management) cells generated by the ABR source and looped back by the destination as backward RM cells, to inform the source of congestion in the network. The network element may directly insert feedback information by adding a value in the Explicit Rate (ER) field of the RM cell. This value may serve as an upper bound on the allowed cell rate (ACR) of the ABR source. The network element may use implicit feedback by setting the EFCI bit in the data cell header of the cells in forward direction. In this case the destination passes this information to the source through backward RM cells. The source uses this information to increase or decrease the ACR subject to the constraint imposed by the ER field. Thirdly, the network element may generate the backward RM cells to explicitly inform the source to decrease or increase the ACR [ATM99] [FEN96].

In [SAI96] it has been shown that it may be difficult to offer high-performance ABR in public networks due to the long round trip time and large number of multiplexed connections. The effectiveness of flow control based on EFCI-marking has been studied. It is observed that ABR with EFCI marking can be supported if peak rate is kept very low. ER-marking switches remove the PCR limitation. Simulation studies performed in [SAI96] show that lowering the PCR for TCP-over-EFCI is more effective.

In general all the flow control schemes based on increase/decrease of traffic flow based on feedback from the network show oscillations. The large amplitude oscillations of pure reactive feedback mechanisms have been studied in [MUK91] [FEN]. In [MUK91] analytical approach has been used to show that the large amplitude variations are possible in the JRJ algorithm for feedback based flow control. The mathematical treatment in [FEN] has been used to prove that such oscillations are controlled to acceptably small magnitudes in a class of algorithms.

The transient behavior of the feedback mechanism has been studied in [BEN98] [KES91]. In [MIS92] it has been shown that hop-by-hop feedback strategies perform far better than end-to-end feedback strategies. While large undamped oscillations have been observed in end-to-end congestion control strategy proposed in [FEN92], it has been shown that in Hop-by-hop scheme proposed in [MIS92] the oscillations are small and they die down fast. No such dampening was observed in the case of TCP.

In [FEN92] it has been shown that adaptive end-to-end based rate control policy performs better than the static rate control policy. The relative improvement in performance of the predictive policy is more for higher values of the correlation coefficient. Discrete time stochastic approach has been used to analyze a generic flow control mechanism. It has been shown that the control mechanism is stable and efficient only if the control actions taken by the source are based on feedback information that includes not only an estimate of the bottleneck rate but also an estimate of the bottleneck queue size.

In [WAN92], a new approach has been presented for deriving quantitative information for rate-based congestion schemes. Using the fluid model for time-dependent  $M(t)/M(t)/1$  queue, the bottleneck capacity has been estimated from round trip delay when the sender is increasing its traffic rate linearly.

Any study of feedback in congestion control in ATM networks has to incorporate both the dynamics of the traffic sources and the relatively large propagation

delay. The impact of large propagation delays has been studied in [MIS92] [WAN91] [MUK91]. However none of these papers consider sources with correlated arrivals. Markov Modulated Fluid sources have been considered in [PAZ95]. The authors have modeled a rate-controlled source with delayed feedback. It is assumed that the switch sends periodic feedback in the form of a single congestion indicator bit. The switch employs a threshold mechanism based on its buffer level to discard excess traffic. The stationary distribution of this system has been obtained using the spectral decomposition.

## 2.5 Problem Overview

The motivation for this thesis comes from the observation that most of the congestion control schemes proposed for ATM Traffic Management were of preventive kind. Though preventive mechanisms like Call Admission Control are in any network that provides guaranteed Quality of Service, preventive mechanisms alone are not enough to safeguard the network from collapsing due to sustained congestion. The reasons for sustained congestion can be many and have already been discussed in Chapter 1.

Thus conceptually what is desirable is a preventive congestion control mechanism like Call Admission Control whose foundations are built upon some kind of reactive flow control mechanism so that not only it eliminate the possibility of sustained congestion but also results in higher resources utilization within the network. The ultimate goal of providing Quality of Service is best achieved with an integrated solution that combines network control that provides feedback for cooperative end-system control [GOY99].

In general, loss probability is defined as the fraction of cells dropped by a queue with a finite buffer size. Though the fraction of cells lost is an important QoS criteria, the quality perceived by the user does depend on the duration for which cell loss takes place. This is especially true in case of the multimedia traffic. The duration of the overload period and its periodicity reflect the load on the network node. Thus a Call Admission Control scheme should include this aspect when accepting or rejecting a call. The focus of this thesis is to model the sojourn time (duration spent) into congested and uncongested state and study its behavior for sources with and without feedback control. This is used to provide a framework so that mean overload period and

underload period may be used to supplement the mechanism for Call Admission Control based on stationary cell loss probability as the QoS criterion.

The aim is to extend the model of sojourn time in order to study the overload behavior under simple binary feedback mechanism with non-negligible propagation delay. Most of the results available in literature for reactive mechanisms are based on traffic, which is not correlated in nature. In this work the effectiveness of feedback mechanism with sources generating traffic which shows some kind of correlation, is being studied. The effectiveness is evaluated using the parameters like mean congestion duration and the ratio of lossy period to non-lossy period and the mean cycle time of congestion periods in the duration of single connection. These parameters depend on traffic characteristics and the traffic load. A study similar to this has been pursued in [PAZ95]. The model employed in [PAZ95] does not take into account the presence of source buffers and hence becomes a special case of the model presented in this work. In [PAZ95] the cell loss performance has been obtained using the stationary probability of system. The work in this thesis concentrates on obtaining the density function for first passage time problem and using it to obtain the performance measures like mean overload and underload periods and mean cycle time of congestion.

For the purpose of simplification the sources have been assumed to be On-Off type with exponentially distributed On and Off periods. In On state the sources generate traffic at a constant rate. No traffic is generated when a source is in Off state. The superimposed arrival process of  $N$  such homogeneous On-Off sources is modeled as a Markov Modulated Rate Process with  $(N+1)$  states.

The sojourn time problem has been studied using the discrete queuing analysis and is well known for its analytical difficulty [LAT93] [LEE92] [LAU00]. In this work the sojourn-time behavior has been studied using the fluid flow model. In Chapter 3, the first passage times for a statistical multiplexer fed by On-Off sources has been modeled using the backward Chapman-Kolmogrov equation. The set of differential equations thus obtained is reduced to matrix differential equation in Laplace Transform domain. This requires solving the eigenvalue problem of a matrix equation in Laplace domain. The properties of the eigenvalues for the matrix-differential equation are obtained and analyzed by extending the work of [ANI82] [REN95] [TAN95]. This approach is used in Section 3.3 for obtaining the solution for first passage time to congestion. These properties are used to segregate the stable eigenvalues from the unstable eigenvalues. The most critical step in this analysis is to identify sets of linear

equations that uniquely determine the probability distributions for the sojourn time at the buffer boundaries. These boundary conditions are used in Chapter 4 to obtain the Laplace transform of probability distribution of sojourn time.

Chapter 4 discusses the procedure to obtain the moments of sojourn time in overload and underload states. A two-state On-Off process has been used to approximate the superposed traffic from N On-Off sources. The results thus obtained are compared with those obtained using simulation. A fast simulator using embedded flow equations developed for the purpose of simulating network of switches has been used for simulation [MUD97]. This simulator has been used to verify the results obtained using the model developed in Chapter 3 and solved in Chapter 4.

In Chapter 5 a simple fluid buffer with deterministic server with rate controlled sources has been used to model the binary feedback mechanism. Feedback with non-negligible feedback delay controls the output rate of the source buffer. This model has been used to study the overload and underload periods in the multiplexer with feedback controlled sources. The fluid flow model has been used to obtain the distribution for the duration of congestion using the sojourn time approach developed in Chapter 3. The performance of feedback mechanism has been studied using infinite source buffer and rate control of source with and without source buffer. The results developed here may be extended for the queue with state dependent output to model the rate control from the downstream node. This extension is useful to model the backpressure mechanism of flow control.

In the end a simple Call Admission Control strategy based on computing the effective bandwidth requirement of all the active connections in the network has been developed. It takes into consideration the buffer availability at the node, the statistical multiplexing of bursty traffic and the QOS requirements. This methodology based on fluid flow analysis of a single server queue served by traffic, which is represented as deterministic flow with random jumps. This approach permits the calculation of asymptotic constant for the buffer occupancy distribution, iteratively. The approach given here leads to more realistic calculation of effective bandwidth than those based on additive effective bandwidth. Another approach based on calculation of asymptotic constant using the instant traffic rate as a continuous random variable has been proposed and results obtained are found to be very close to those obtained by the first method. In the end, the effective bandwidth of the combined traffic at an ATM multiplexer is obtained using the concept of equivalent source. This two state

equivalent source results in the same overload and underload behavior at an ATM multiplexer as that due to superposed traffic from  $N$  sources.

Chapter 7 summarizes the results and discusses the scope of future work.

## 2.6 Summary

Analytical study of any network requires modeling of different components like input arrival process, service time distribution and queuing policy. Any tractable analysis makes certain simplifying assumptions. The Literature Survey gives a brief introduction to these aspects of analytical study describing various approximations that simplify a study of network. It also gives the conditions under which these approximations are valid. Study deals with source modeling, multiplexer model, traffic management mechanisms and feedback control. The Chapter concludes with the problem overview.

## Chapter 3

# Sojourn Time Analysis

Traditionally packet/cell loss probability has been used as a parameter for QoS performance measure of a packet switched network. It has also been used as an important parameter in CAC (Call Admission Control) procedures proposed in the literature. The decision to admit a connection is based on whether the packet loss probability of the new connection request as well as that of existing connections can be met or not. Almost all the proposed CAC procedures in literature use the model of stationary behavior of a packet multiplexer for obtaining the packet/cell loss probability. Though packet/cell loss behavior of a statistical multiplexer is an important measure for providing QoS guarantees, the real-time multimedia traffic is less sensitive to small percentage of packet/cell loss. Recent developments in multimedia coding, e.g. MPEG-4 coding have even incorporated error resilience against network errors. For multimedia services like multimedia streaming, it is the duration of the overload period/congestion duration that has direct impact on the QoS perceived by a user. To understand this impact on QoS, we need adequate models to describe overload periods/congestion duration. In this work, sojourn-time analysis has been used to obtain the expressions for probability density function for the length of the overload and underload periods in a statistical multiplexer. As discussed in Section 2.5, the sojourn time problem has been studied in earlier works [LAT93] [LEE92] [LAU00] using the discrete queuing analysis and is well known for its analytical difficulty. The fluid flow approach used in this chapter to model the probability density function for overload and underload periods leads us to more tractable solutions.

The first passage times to transition from overload state to underload state is obtained using the embedding technique [BEL76]. This method leads to partial differential equations. By making use of Laplace transform, these partial differential equations are reduced to the eigenvalue problem of a matrix equation in the Laplace transform domain. These equations are found to be similar to those obtained for steady state statistical multiplexer in [ANI82] and for transient solution of buffer behavior in statistical multiplexer in [REN95]. The characteristics of eigenvalues for the set of

matrix equations for the density for first passage time is obtained using the approach followed in [ANI82] and [REN95].

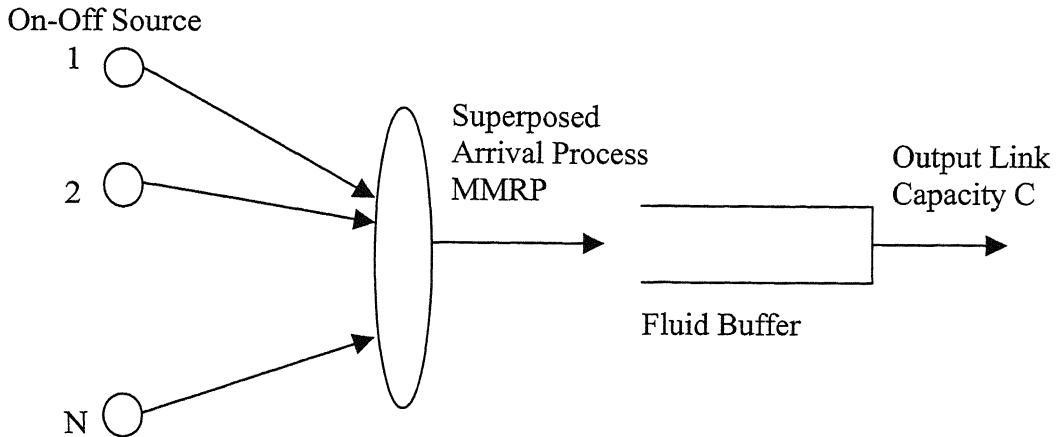
The most critical aspect of obtaining the solution for the density functions for overload and underload periods, is the identification of boundary conditions and using the characteristics of the eigenvalues along with boundary conditions to obtain the constant matrix. The solution obtained in Chapter 3 and Chapter 4 is applied to obtain the overload and underload periods for a statistical multiplexer with binary feedback controlled sources in Chapter 5.

A fluid buffer model is described in Section 3.1 along with the justification for fluid flow approach. Section 3.2 develops the matrix differential equation for the density function of first passage time to reach the state of congestion. The boundary conditions and eigenvalue characteristics are in Section 3.3. Probability density function for first passage time to underload state is obtained in Section 3.4. Expressions for sojourn times into overload/congestion duration and underload periods are given in Section 3.5. The application of boundary condition to the expressions obtained in Section 3.3 and 3.4 is covered in Chapter 4. Numerical examples are provided in Chapter 4 along with the simulation procedure used to validate the results obtained using the analysis described in this chapter.

### 3.1 Fluid Buffer Model

The model used here is of a statistical multiplexer fed by On-Off type sources. The statistical nature of multiplexing implies that there are periods when the input traffic to the multiplexer is greater than the output link capacity. This results in queuing and may lead to buffer overflow. Thus there are periods when the multiplexer is said to be in congestion. It is assumed that during a single connection duration, a multiplexer alternates between overload state and underload state. This assumption is valid in the case of a heavily loaded node. The exact definition of overload/congestion duration and underload period is given later in Section 3.1.

On-Off source generates fluid traffic at a constant rate in On state and no traffic is generated in off state. The aggregate fluid arrival at the input of a statistical multiplexer is modeled as a Markov Modulated Rate Process (MMRP) (Figure 3.1). The MMRP has briefly been discussed in Section 2.1 and is once again described in detail later in this section. At any instant the aggregate fluid arrival at the input of a multiplexer depends on the state of the modulating Markov Process.



**Figure 3.1 A Fluid Buffer Model**

In this thesis, the fluid flow approach has been used for modeling of a statistical multiplexer. In a traditional queuing approach the arrivals are discrete events, whereas in the fluid flow approach the arrival entity is continuous and is referred to as fluid. This approach has been traditionally used to model the behavior of dam and fluid reservoirs. In last few decades interest has been shown in using this approach for modeling queues and queuing networks using fluid flow approach [ANI82] [KOS74].

The choice of fluid flow model was governed by two reasons. One was that traditional queuing approach results in what is known as curse of dimensionality. When analyzing the modern day network queues fed by doubly stochastic processes like Markov Modulated Poisson Processes or Batch Markov Arrival Processes, the state space blows up with increase in number of sources and buffer size. Thus it is very difficult to obtain results for meaningful large systems. This problem is reduced by one dimension if fluid flow model is used. This allows the usage of fluid flow approach for analyzing large system.

The convenience of ATM networks with small sized packets called cells was another reason for the choice of fluid flow model for modeling network queues. In ATM, the cell inter-arrival time and cell service time are very small in comparison with the duration of the burst. The time scale of interest in this thesis is burst level phenomenon in the arrival process. Hence the cell arrival process can be seen as a continuous flow of fluid. Fluid flow approach does not take into account the variation in inter cell arrival interval. The stochastics associated with the ~~inter-cell arrival process~~ TCSAT  
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is being ignored as it does not contribute to the queuing behavior in large buffer queues [NOR91] [CHO97].

The fluid flow approach results in more tractable analytical model than the actual queuing system. Queuing approach provides advantage in cases where the advantage of structural properties of the one-step transition matrices allows development of more efficient algorithms. Though this advantage is not there in fluid flow approach but the tractability of fluid models and diffusion processes in terms of Laplace Transforms is well known.

In the fluid flow model the input fluid process is stochastic in nature where the arrival rate  $r(t)$  has an associated probability measure. The fluid flows into a reservoir that may be of finite or infinite size (Figure 3.1). This reservoir will be called the fluid buffer throughout this thesis. The fluid buffer empties through one or many outlets. The outlet rate may be constant or stochastic. This generic model is used for analyzing the behavior of statistical multiplexer and other buffers in this thesis.

The arrival fluid rate  $r(t)$  is modulated by a background stochastic process. This modulating process governs the rate at which the fluid flows into the fluid buffer. In present work the modulating process is assumed to be a finite Markov chain, unless specified otherwise. The instantaneous fluid generation rate from such a source is dependent on the state of the modulating Markov chain. This arrival process is called a Markov Modulated Rate Process.

Let  $N + 1$  be the number of states in the background Markov chain  $M(t)$  defined over state space  $[0, N]$ . When the modulating process is in state  $i \in [0, N]$ , the fluid is generated at a rate  $r_i$ . Let  $X(t)$  represent the quantity of fluid in fluid buffer at time  $t$ . The fluid flows out of the fluid buffer at a constant rate  $C$ . Hence the output rate from the buffer  $R_o(t)$  is given by

$$R_o(t) = \begin{cases} C & X(t) > 0 \text{ or } r_i(t) \geq C \\ r_i(t) & \text{otherwise} \end{cases} \quad (3.1)$$

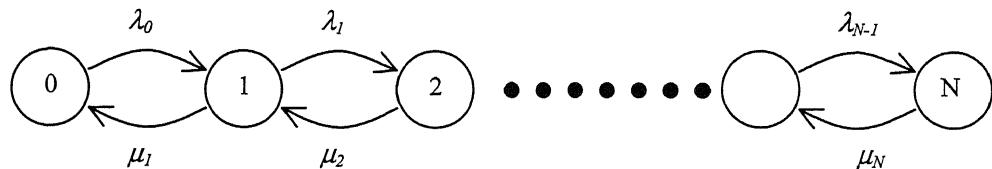
Throughout this thesis the Markov Modulated Rate Process defining the fluid arrival at the fluid buffer is obtained from the superposition of  $N$  homogeneous, independent On-Off fluid sources. As mentioned earlier, an On-Off source generates fluid at a constant rate  $R$  when in On state and no fluid is generated in Off state. The On and Off periods are independent and are assumed to be exponentially distributed unless specified otherwise.

### **3.1.1 On-Off Source Description**

|                         |             |
|-------------------------|-------------|
| Peak Rate               | $R$         |
| Mean Source Utilization | $\rho$      |
| Mean Rate               | $\rho R$    |
| Mean On period          | $1/\mu$     |
| Mean Off period         | $1/\lambda$ |

The source utilization factor  $\rho$  is defined as

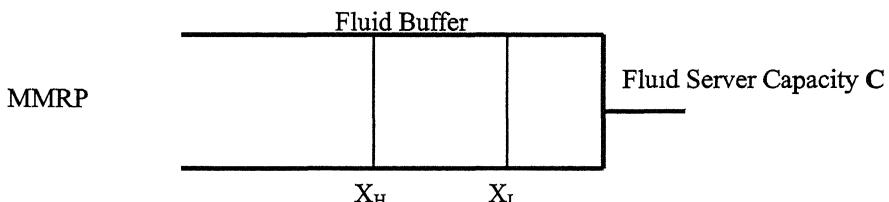
$$\rho = \frac{\lambda}{\lambda + \mu} \quad (3.2)$$



**Figure 3.2 Background Birth and Death Process**

The combined arrival process from  $N$  homogeneous independent On-off sources is a Markov Modulated Rate Process with the modulating Markov Chain being defined by a Birth and Death process (Figure 3.2). When the modulating process is in state  $i$ , equivalent to  $i$  sources being in On state, the fluid arrival rate is given by

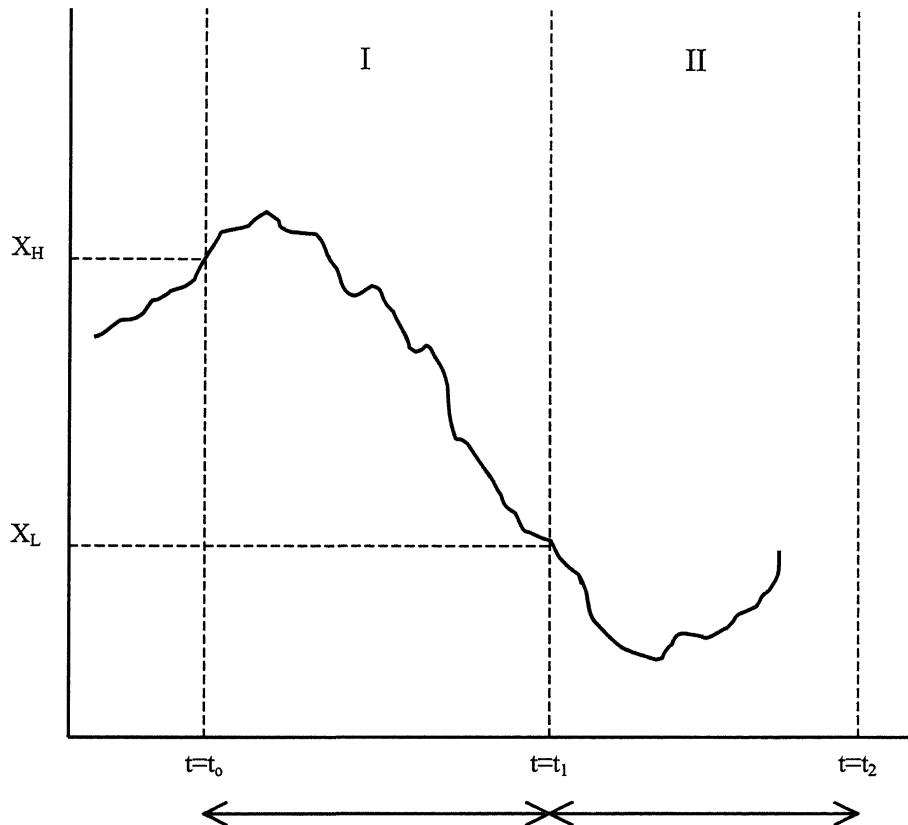
$$r_i(t) = iR \quad i \in \{0, N\} \quad \text{--- (3.3)}$$



**Figure 3.3 Single Server Fluid Buffer with Two Thresholds**

A two level threshold of fluid buffer (Figure 3.3) is used for defining the duration for which the system remains in congestion. Let these levels be called high threshold  $X_H$  and low threshold  $X_L$ . The multiplexer is in congestion between the time when the fluid buffer crosses the high threshold from below, to the instant when the

fluid buffer contents fall below the low threshold  $X_L$  for the first time since reaching the congested state. In Figure 3.4 the duration between  $t = t_0$  and  $t = t_1$  is the period in which the fluid buffer is in congested state. The multiplexer is said to be in uncongested state in period between  $t = t_1$ , when the buffer content fall below  $X_L$  and the instant  $t = t_2$  when the fluid buffer crosses the high threshold  $X_H$  from below for the first time. This definition of congestion duration and uncongested period is used throughout in this work. The fluid buffer is assumed to be infinite. When modeling multiplexer with finite buffer, the buffer size is assumed to be  $X_B$ .



**Figure 3.4 Buffer Content Variation with Time**

### 3.2 First Passage Time To Reach Congestion

Let us consider a multiplexer with finite or infinite buffer fed by a source defined by Markov Modulated Rate Process. At any instant the multiplexer is either in the congested state or in uncongested state. In order to obtain the moments of time spent in each state, the probability density for the first passage time need to be obtained. In this section the model for the first passage time for the multiplexer to reach

the state of congestion given that it was in uncongested state at time  $t=0$  has been obtained. Since the multiplexer buffer content remain below the high-threshold level for the duration of interest, the set of equations used to model the densities for the first passage time remain the same for infinite as well as finite buffered multiplexer.

The first passage time to congestion has been obtained by using the backward Chapman-Kolmogrov equation. The method based on backward Chapman-Kolmogrov equation, leads to Matrix differential equations that are solved by obtaining the spectrum of the key matrix and applying the boundary conditions.

Let  $X(t)$  be the random variable representing the content of the fluid buffer with an initial value of  $X(0)=x$  at time  $t=0$ . The fact that the multiplexer is in uncongested state implies that  $0 \leq x \leq X_H$ . Let the phase of the modulating process  $M(t)$  be  $i$  at time  $t = 0$ . The process  $X(t)$  has two barriers, one at  $X=0$  and another at  $X=X_H$ . We are interested in finding the probability density of the first passage time to crossing the barrier at  $X=X_H$  with any number of buffer emptiness. To find this we define

$p_{i,j}(x,t)dt \equiv$  The probability that the high threshold  $X_H$  is crossed for the first time from below between time  $t$  and  $t+\Delta t$  given that any number of emptiness of fluid buffer occur in the time interval  $(0,t)$  and that  $X(0)=x$ ,  $0 \leq x < X_H$  at time  $t=0$ . At time  $t=0$  The phase of the modulating process  $M(t)$  was  $i$  at time  $t=0$  and is  $j$  at  $t=t$ .

Using the definition given above, the system of partial differential equations governing the dynamics of the first passage time are obtained next. For this we consider the different independent possibilities which are mutually exclusive in the interval  $(0,\Delta t)$ . Then proceeding to limit and as  $\Delta t$  tends to zero, we arrive at the imbedding equation for  $p_{i,j}(x,t)$ .

$$p_{i,j}(x,t + \Delta t) = \lambda_i \Delta t p_{i+1,j}(x + (r_i - C)\Delta t, t) + \mu_i \Delta t p_{i-1,j}(x + (r_i - C)\Delta t, t) + (1 - \lambda_i \Delta t - \mu_i \Delta t) p_{i,j}(x + (r_i - C)\Delta t, t) \quad x \geq 0, t \geq 0$$

(3.4)

Here  $\lambda_i$  and  $\mu_i$  are birth and death rates of the modulating birth and death process when the process is in state  $i$ . Subtracting  $p_{i,j}(x,t)$  from both the sides in above equation and after rearranging and dividing by  $\Delta t$  and taking the limit as  $\Delta t \rightarrow 0$ , we get

$$\frac{\partial p_{i,j}(x,t)}{\partial t} - (r_i - C) \frac{\partial p_{i,j}(x,t)}{\partial x} = -(\lambda_i + \mu_i)p_{i,j}(x,t) + \lambda_i p_{i+1,j}(x,t) + \mu_i p_{i-1,j}(x,t) \quad 0 \leq i, j \leq N, x \geq 0, t \geq 0 \quad (3.5)$$

The above equation may be written in the more compact form of matrix differential equations. Let  $p(x,t)$  be an  $(N+1) \times (N+1)$  probability density matrix of the first passage time to congestion given that the initial content in fluid buffer was  $x$ . The  $ij^{\text{th}}$  element of the two dimensional matrix  $p(x,t)$  is given by the probability density function  $p_{i,j}(x,t)$  of the first passage time to the congestion state as defined above.

$$p(x,t) \equiv \begin{bmatrix} p_{0,0}(x,t) & p_{0,1}(x,t) & \dots & p_{0,N}(x,t) \\ p_{1,0}(x,t) & p_{1,1}(x,t) & \dots & p_{1,N}(x,t) \\ \vdots & \vdots & \ddots & \vdots \\ p_{N-1,0}(x,t) & \dots & \dots & p_{N-1,N}(x,t) \\ p_{N,0}(x,t) & p_{N,1}(x,t) & \dots & p_{N,N}(x,t) \end{bmatrix} \quad (3.6)$$

The drift  $D$  as the diagonal matrix of size  $(N+1) \times (N+1)$  matrix as given below.

$$D \equiv \begin{bmatrix} r_0 - C & 0 & \dots & 0 \\ 0 & r_1 - C & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & 0 & r_N - C \end{bmatrix} \quad (3.7)$$

Let  $M$  be the transition rate matrix of the modulating continuous time ergodic Markov Process  $M(t)$ . Then the transition matrix  $M$  for birth and death modulating process is given by

$$M \equiv \begin{bmatrix} -\lambda_0 & \lambda_0 & \dots & \dots & 0 \\ \mu_1 & -\lambda_1 - \mu_1 & \lambda_1 & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & \dots & \dots & \lambda_{N-1} \\ 0 & \dots & \dots & \mu_N & -\mu_N \end{bmatrix} \quad (3.8)$$

If  $\pi$  is the stationary probability vector for the states of the Markov Process  $M(t)$  and is given by:

$$\pi = [\pi_0 \ \pi_1 \ \dots \ \pi_N] \quad (3.9)$$

Then  $\pi$  and  $M$  satisfy the following relationship

$$\pi \cdot M = 0 \quad (3.10)$$

Using the above definitions, the partial differential equations for the first passage time probability densities are reduced to the Matrix differential equation of the form:

$$\frac{\partial p(x,t)}{\partial t} - D \frac{\partial p(x,t)}{\partial x} = Mp(x,t) \quad 0 \leq x \leq X_H, t \geq 0 \quad (3.11)$$

The solution to this equation is obtained using the Laplace transform approach and obtaining the eigenvalues and eigenvectors for the key matrix thus obtained. The procedure involved in obtaining the solution is discussed in next section.

### 3.3 Solution for First Passage Time to Congestion

Let  $\tilde{p}(x,s)$  be the Laplace transform of  $p(x,t)$  with respect to parameter  $t$  given by:

$$\tilde{p}(x,s) = \int_0^{\infty} e^{-st} p(x,t) dt \quad s > 0 \quad (3.12)$$

Now taking Laplace transform with respect to  $t$ , (3.11) may then be written as

$$D \frac{\partial \tilde{p}(x,s)}{\partial x} = (sI - M) \tilde{p}(x,s) - p(x,0) \quad (3.13)$$

where  $I$  is the  $(N+1) \times (N+1)$  identity matrix. The above equation is a first order matrix differential equation. The factor  $p(x,0)$  in Equation 3.13 is an  $(N+1) \times (N+1)$  probability density matrix for the first passage time to congestion being zero. This is only possible if the fluid buffer content is equal to high threshold value  $X_H$  at time  $t=0$ , and the phase of the modulating Markov Process is such that there is a positive input drift (phase is  $i$  such that  $r_i > C$ ). Hence for all  $i, j \in [0, N]$ , the element  $p_{i,j}(x,0)$  of  $p(x,0)$  is given by:

$$p_{i,j}(x,0) = \begin{cases} \delta(x - X_H) & i = j, \text{ } i \text{ s.t. } r_i > C, \text{ } x \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (3.14)$$

Here  $\delta(x - X_H)$  is a unit impulse at  $x = X_H$ . Now consider an  $(N+1) \times (N+1)$  diagonal matrix  $z(s)$  such that each diagonal element  $z_i(s)$  of the diagonal matrix is an eigenvalue of the key matrix  $D^{-1}(sI - M)$ . Let  $V(s)$  be a row vector of order  $(N+1)$  such that it's each element  $v_i(s)$  is the right eigenvector for the  $i$ th eigenvalue  $z_i(s)$  of the key matrix  $D^{-1}(sI - M)$ . Since  $z(s)$ ,  $V(s)$  and key matrix  $D^{-1}(sI - M)$  have been defined to satisfy the following equation,

$$V(s)z(s) = D^{-1}(sI - M)V(s) \quad (3.15)$$

The solution to Equation 3.13 may be written as

$$\tilde{p}(x, s) = V(s)e^{z(s)x}A(s) - V(s)e^{z(s)(x-X_H)}V^{-1}(s)D^{-1}L_H \quad (3.16)$$

where  $A(s)$  is the constant coefficient matrix of size  $(N+1) \times (N+1)$ .  $L_H$  is also a  $(N+1) \times (N+1)$  diagonal matrix with only the elements corresponding to  $i \neq r_i > C$  being unity. The probability density for the first passage time to congestion may be given by Equation 3.16. This requires obtaining the eigenvalues, corresponding right eigenvectors and using the boundary conditions for finding the constant coefficient  $a_y(s)$ , elements of constant coefficient matrix  $A(s)$ .

### 3.3.1 Eigenvalues and Eigenvectors

If the superposed arrival process has  $N+1$  states then let  $N_1$  be the number of states which result in positive drift ( $r_i > C$ ) in the statistical multiplexer and  $N_2$  be the number of states with negative drift, such that

$$N_1 + N_2 = N + 1 \quad (3.17)$$

For the system where the combined arrival process is superposition of  $N$  homogeneous On-Off sources, each with peak rate  $R$ , the drift parameters and the parameters of infinitesimal generator  $M$  are given by

For  $i \in [0, N]$

$$\begin{aligned} r_i &= iR \\ \lambda_i &= (N - i)\lambda \\ \mu_i &= i\mu \end{aligned} \quad (3.18)$$

Let  $z$  be an eigenvalue matrix of  $D^{-1}(sI - M)$  and  $W = [w_0 \quad \dots \quad w_N]^T$  be the corresponding left eigenvector matrix. Each element  $w_i$  of the left eigenvector

matrix is a row vector of size  $N+1$  given by  $w_i = [w_{i0} \quad \dots \quad w_{iN}]$ . Then the eigenvalues for the key matrix  $D^{-1}(sI - M)$  are obtained using the following equation:

$$zDW = W(sI - M) \quad (3.19)$$

If  $W_i(Z)$  is the generating function for  $w_i$ , and  $W_i'(Z)$  its first derivative defined by

$$W_i(Z) \equiv \sum_{j=0}^N w_{ij} Z^j \quad (3.20)$$

$$W_i'(Z) = \sum_{j=1}^N j w_{i,j} Z^{j-1} \quad (3.21)$$

Using the parameters of  $D$  and  $M$  as given by Equation 3.18 along with Equation 3.20 and Equation 3.21 in Equation 3.19, the following equations in  $W_i(x)$  and  $W_i'(x)$  is obtained for all  $i$ ,  $0 \leq i \leq N$ .

$$\begin{aligned} z_i R x W_i'(Z) - z_i C W_i(Z) &= (s + N\lambda) W_i(Z) + (\mu - \lambda) Z W_i(Z) - N\lambda Z W_i(Z) + \\ &\quad \lambda x^2 W_i'(Z) - \mu W_i'(Z) \end{aligned} \quad (3.22)$$

$$\frac{W_i'(Z)}{W_i(Z)} = \frac{N\lambda(Z-1) - z_i C - s}{\lambda Z^2 + (\mu - \lambda - z_i R)Z - \mu} \quad (3.23)$$

Let  $Z_0$  and  $Z_1$  be the two roots of the quadratic in the denominator of the right-hand side given by:

$$Z_0 = \{-(\mu - \lambda - z_i R) + \sqrt{(\mu - \lambda - z_i R)^2 + 4\lambda\mu}\} / 2\lambda \quad (3.24)$$

$$Z_1 = \{-(\mu - \lambda - z_i R) - \sqrt{(\mu - \lambda - z_i R)^2 + 4\lambda\mu}\} / 2\lambda \quad (3.25)$$

Thus Equation 3.23 may be written as

$$\frac{W_i'(Z)}{W_i(Z)} = \frac{c_1}{Z - Z_0} + \frac{N - c_1}{Z - Z_1} \quad (3.26)$$

The constant  $c_1$  is obtained by comparing the terms in Equation 3.23 with those in Equation 3.26.

$$c_1 = \frac{N\lambda(Z_0 - 1) - z_i C - s}{\lambda(Z_0 - Z_1)} \quad (3.27)$$

The solution for Equation 3.26 may be given as

$$W_i(Z) = (Z - Z_0)^{c_1} (Z - Z_1)^{(N - c_1)} \quad (3.28)$$

Since by definition  $W_l(Z)$  is a polynomial in  $Z$  of degree  $N$  and since  $Z_0$  and  $Z_1$  are distinct, it implies that  $c_l$  is an integer in  $[0, N]$ . Let this integer be represented by  $l$ . Substituting the value of  $Z_0$ ,  $Z_1$  in Equation 3.27, following set of quadratics in the unknown eigenvalue  $z_i$  are obtained

$$a(l, s)z_i^2 + b(l, s)z_i + c(l, s) = 0 \quad l \in [0, N] \quad (3.29)$$

where the constants  $a(l, s)$ ,  $b(l, s)$  and  $c(l, s)$  are given by

$$a(l, s) = \left(\frac{N}{2} - l\right)^2 R^2 - \left(\frac{N}{2}R - C\right)^2 \quad (3.30)$$

$$b(l, s) = -2(\mu - \lambda)\left(\frac{N}{2} - l\right)^2 R + 2\left(\frac{N}{2}R - C\right)\left[\frac{N}{2}(\mu + \lambda) + s\right] \quad (3.31)$$

$$c(l, s) = (\mu + \lambda)^2\left(\frac{N}{2} - l\right)^2 - \left[\frac{N}{2}(\mu + \lambda) + s\right]^2 \quad (3.32)$$

Hence we can say that there will be  $(N+1)$  distinct roots for the above given set of quadratic equations. The eigenvalues for  $l \in [0, N]$  are given by:

$$z_i = \frac{\left(\frac{N}{2} - l\right)}{R} - \frac{\left(\frac{N}{2}R - C\right)[sR + \mu C + \lambda(NR - C)]}{(NR - IR - C)(C - IR)R} + \frac{\left(\frac{N}{2} - l\right)\sqrt{(sR + \mu C + \lambda(NR - C))^2 + 4\mu\lambda(NR - IR - C)(IR - C)}}{(NR - IR - C)(C - IR)} \quad (3.33)$$

Let there be  $N_1$  positive eigenvalues and  $N_2$  negative eigenvalues as obtained from Equation 3.33 such that  $N_1 + N_2 = N + 1$ . Since the probability density for the first passage time is piecewise continuous and bounded hence from the existence theorem for Laplace transform, the transform of probability density of first passage time to congestion must exist for all  $s$  greater than some finite constant. This is only possible if the coefficients corresponding to the exponential terms with positive exponents in the solution (Equation 3.16) are zero. Hence the positive and negative eigenvalues need to be segregated in order to obtain the solution for the first passage time.

To find whether an eigenvalue is positive or negative, the sign of an eigenvalue  $z_i(s)$  at  $s=0$  is observed. It is also found from the derivative of  $z_i(s)$  that the eigenvalues are either strictly increasing or strictly decreasing in right half of the  $s$ -plane (Appendix 1). The following summarizes the properties of the eigenvalues:

1. For  $l$  varying from 0 to  $N$  the above given set of quadratic equations will result in  $(N+1)$  non-zero, distinct roots.
2. There are  $\lfloor C/R \rfloor + 1$  negative roots at  $s=0$  and these are strictly decreasing function of  $s \geq 0$
3. There are  $N - \lfloor C/R \rfloor - 1$  positive roots at  $s=0$  and each are strictly increasing function of  $s \geq 0$
4. There is one root  $z=0$  at  $s=0$  and it is strictly increasing function of  $s \geq 0$

There exists a one-to-one correspondence between positive diagonal elements of  $D$  and the eigenvalues of  $D^{-1}(sI - M)$  being positive. Similarly there is also a one-to-one correspondence between negative diagonal elements of the drift matrix  $D$  and the negative eigenvalues of  $D^{-1}(sI - M)$ .

The left eigenvector  $w_i$  for the eigenvalue  $z_i(s)$  for all values of  $i \in [0, N]$  is obtained from the corresponding generating function  $W_i(x)$ . There will be  $N+1$  such generating functions corresponding to each of the eigenvalues. Let  $z_i(s)$  be an eigenvalue corresponding to a particular value of  $l$  in Equation 3.33. Using this value of  $z_i(s)$  in Equation 3.24 and Equation 3.25,  $Z_0$  and  $Z_1$  are obtained. The  $j^{\text{th}}$  component of the eigenvector  $w_i$  is given by:

$$w_{ij} = (-1)^{N-j} \sum_{k=0}^l \binom{l}{k} \binom{N-l}{j-k} Z_0^{l-k} Z_1^{N-l-j+k} \quad 0 \leq j \leq N \quad (3.34)$$

The corresponding right eigenvector  $v_i$  is obtained by making use of the properties of the matrix  $M$  for the birth and death process (Appendix 2) and is given by

$$v_i = \tau^2 w_i^T \quad (3.35)$$

where  $\tau$  is the diagonal matrix with the  $i^{\text{th}}$  diagonal element given by

$$\tau_i = \sqrt{(\lambda/\mu)^i \binom{N}{i}} \quad (3.36)$$

and  $w_i^T$  is transpose of left eigenvector  $w_i$ .

### 3.3.2 Boundary Conditions

The constant matrix  $A(s)$  in Equation 3.16 is obtained in Section 4.1 using the boundary conditions defined here. The first passage time to congestion is independent of the size of fluid buffer. Hence the same boundary conditions apply to finite as well

as infinite size fluid buffer. The first passage density is constrained by two boundaries at  $x=0$  and  $x=X_H$ .

By definition of first passage time to congestion it is clear that there is no possibility of the fluid buffer crossing the high threshold if the state of the modulating source  $j$  at time  $t$  is such that the net drift is negative. Hence it can be said that

$$p_{i,j}(x,t) = 0 \quad j \text{ s.t. } r_j \leq C, \quad t \geq 0$$

and also

$$\tilde{p}_{i,j}(x,s) = 0 \quad j \text{ s.t. } r_j \leq C \quad (3.37)$$

Another boundary condition of interest is at  $x = X_H$ . If the state at time  $t=0$  of the modulating process is such that the inflow is greater than the outflow, then the high threshold will be crossed at  $t=0$  with probability 1.

Third boundary condition is defined at  $x = 0$ . Using the first principle (Equation 3.4) to obtain the governing equations for the probability density of first passage time to congestion given that the fluid contents were zero at time  $t=0$  gives the following:

$$\begin{aligned} \frac{\partial p_{i,j}(0,t)}{\partial t} &= -(\lambda_i + \mu_i)p_{i,j}(0,t) + \lambda_i p_{i+1,j}(0,t) \\ &\quad + \mu_i p_{i-1,j}(0,t) \quad 0 \leq i \leq \lfloor C/R \rfloor, 0 \leq j \leq N, t \geq 0 \end{aligned} \quad (3.38)$$

$$\Rightarrow \frac{\partial p_{i,j}(0,t)}{\partial x} = 0 \quad 0 \leq i \leq \lfloor C/R \rfloor, 0 \leq j \leq N, t \geq 0 \quad (3.39)$$

The procedure to use these boundary conditions in order to obtain the constant matrix  $A(s)$  in Equation 3.16 is described in Chapter 4.

The density for the first passage time to congestion itself is obtained by numerically inverting the Laplace of the probability density function. If  $\pi(0) = [\pi_0(0) \dots \pi_N(0)]$  is the initial probability vector giving the probability that the modulating chain was in state  $i$  at time  $t=0$ , then the probability density for the first passage time to congestion is given by

$$p_x(t) = \pi(0)p(x,t).e \quad (3.40)$$

where  $e$  is the unit column matrix.

Next section develops the equations governing the duration for which the multiplexer remains in congestion by considering it as a first passage time problem.

### 3.4 Congestion Duration

As defined earlier, the statistical multiplexer represented by the fluid buffer with constant rate server is said to enter a state of congestion if the fluid contents in the buffer exceed the high threshold value. The system remains in the congested state until the fluid contents fall below the low threshold value. The time required for the contents of the fluid buffer to fall below the low threshold level for the first time given that at time  $t=0$  the system was in congested state, is called congestion duration.

The imbedding technique is again used to obtain the set of equations governing the duration for which the system remains in the state of congestion. The fluid buffer is assumed to be of infinite size. Let  $X(t)$  be the random variable representing the content of the fluid buffer with an initial value of  $X(0)=x$  at time  $t=0$ . The fact that the multiplexer is in congested state implies that at some point of time the fluid buffer had crossed the high threshold and at time  $t=0$  the buffer content is greater than the low threshold  $X_L$ . Let the phase of the modulating process  $M(t)$  be  $i$  at time  $t=0$ . The probability density of the first passage time to crossing the barrier at  $X=X_L$  defined as:

$p_{i,j}(x,t)dt \equiv$  The probability that the low threshold  $X_L$  is crossed for the first time from above between  $t=t$  and  $t=t+\Delta t$  and that  $X(t) = x$  at time  $t=0$ . At time  $t=0$  the modulating process  $M(t)$  was in phase  $i$  and is in phase  $j$  at time  $t$ .

The system of partial differential equations is obtained by considering the different independent possibilities which are mutually exclusive in the interval  $(0, \Delta t)$ . Using the same approach as the one for first passage time to congestion, the probability density function for first passage time to underload state,  $p_{i,j}(x,t)$ , is governed by the following set of partial differential equations. Proceeding to limit as  $\Delta t$  tends to zero the imbedding equation for  $p_{i,j}(x,t)$  is given by

$$\frac{\partial p_{i,j}(x,t)}{\partial t} - (r_i - C) \frac{\partial p_{i,j}(x,t)}{\partial x} = -(\lambda_i + \mu_i)p_{i,j}(x,t) + \lambda_i p_{i+1,j}(x,t) + \mu_i p_{i-1,j}(x,t) \quad 0 \leq i, j \leq N, x \geq X_L, t \geq 0 \quad (3.41)$$

where  $\lambda_i$  and  $\mu_i$  are birth and death rates of the modulating birth and death process when the process is in state  $i$ .

The above given set of equations are represented in a compact form using the definitions of drift matrix  $D$  and rate transition matrix  $M$  for the modulating process, as defined in Section 3.2. Thus the partial differential equations for the first passage time probability densities are given by:

$$\frac{\partial p(x,t)}{\partial t} - D \frac{\partial p(x,t)}{\partial x} = Mp(x,t) \quad x \geq X_L, t \geq 0 \quad (3.42)$$

This equation for the first passage time to underload state is same as the one obtained for the first passage time to congestion and hence will be solved using the same approach as that followed in Section 3.2. The solution for the matrix differential equation (Equation 3.42) is obtained using the Laplace transform approach. Let  $\tilde{p}(x,s)$  be the Laplace transform of  $p(x,t)$  with respect to parameter  $t$ .

$$\tilde{p}(x,s) = \int_0^{\infty} e^{-st} p(x,t) dt \quad s > 0 \quad (3.43)$$

Then Equation 3.42 is given by

$$D \frac{\partial \tilde{p}(x,s)}{\partial x} = (sI - M) \tilde{p}(x,s) - p(x,0) \quad x \geq X_L \quad (3.44)$$

where  $I$  is the  $(N+1) \times (N+1)$  identity matrix. The factor  $p(x,0)$  gives the probability of sojourn time into congestion being zero. This is only possible if at time  $t=0$  the contents of fluid buffer  $X(t)=X_L$  and the phase of the modulating process  $M(t)$  is such that  $r_i < C$ . The first passage time density  $p(x,0)$  is an  $(N+1) \times (N+1)$  matrix with it's element  $p_{i,j}(x,0)$  given by the following equation

$$p_{i,j}(x,0) = \begin{cases} 0 & i \neq j, 0 \leq i, j \leq N \\ 0 & i = j, \text{ s.t. } r_i \geq C \\ \delta(x - X_L) & i = j, \text{ s.t. } r_i < C \end{cases} \quad (3.45)$$

The solution to the matrix differential equation (Equation 3.44) is obtained as

$$\tilde{p}(x,s) = V(s)e^{z(s)x} A(s) - V(s)e^{z(s)(x-X_L)} V^{-1}(s) D^{-1} L_L \quad (3.46)$$

where  $A(s)$  is an  $(N+1) \times (N+1)$  matrix of constant coefficients  $a_{ij}(s)$ .  $L_L$  is an  $(N+1) \times (N+1)$  diagonal matrix. It has zero elements except for the unit diagonal elements corresponding to index  $i$  such that  $r_i < C$ . The diagonal matrix  $z(s)$  is the eigenvalue matrix for the key matrix  $D^{-1}(sI - M)$ . The similarity in the matrix differential equation and solution for the probability density for the first passage time to

congestion and the congestion duration means that the eigenvalues and eigenvectors of the key matrix have the same properties. The eigenvalues are given by Equation 3.33 and corresponding left eigenvectors by Equation 3.34.

The boundary conditions required for obtaining the constant coefficient matrix  $A(s)$  is discussed next. The first passage time to underload state will depend on the size of fluid buffer. The boundary conditions will be different for finite sized buffer and infinite size buffer. For both finite and infinite buffer size, the density function for first passage time to underload state experiences a boundary at  $x = X_L$ .

The fluid buffer contents reduce below the low threshold at between time  $t$  and  $t=t+\Delta t$  if and only if the phase  $j$  of the modulating process is such that there is a negative drift (i.e.  $r_j < C$ ). Hence it can be said that for both finite and infinite buffer size

$$p_{i,j}(x,t) = 0 \quad j \text{ s.t. } r_j \geq C, \quad t \geq 0 \quad (3.47)$$

and also

$$\tilde{p}_{i,j}(x,s) = 0 \quad j \text{ s.t. } r_j \geq C \quad (3.48)$$

If the initial buffer content at time  $t=0$  is  $X_L$  and the state of the modulating process is such that the inflow is less than the outflow, then the low threshold  $X_L$  will be crossed at  $t=0$  with probability 1.

In case of finite buffer of size  $X_B$ , if the initial contents of fluid buffer at time  $t=0$  are given by  $x = X_B$  and the net inflow is more than the net outflow, then the probability density for the first passage time to underload state satisfies the following boundary condition.

$$\frac{\partial p_{i,j}(X_B, t)}{\partial x} = 0 \quad [C/R] + 1 \leq i \leq N, \quad 0 \leq j \leq N, \quad t \geq 0 \quad (3.49)$$

In case of infinite buffer, the boundary condition given by Equation 3.49 is not applicable. The function  $p_{i,j}(x, t)$  represents the probability density function for the first passage time. Since the integral of  $p_{i,j}(x, t)$  over all  $t$  on real axis  $[0, \infty)$  is bounded by unity, hence the Laplace of  $p_{i,j}(x, t)$  is also convergent. This requires that the stable solution of Equation 3.46 should not be divergent. This implies that the coefficients of

the exponential terms in Equation 3.46 with positive exponents that increase with increasing  $s$ , will be zero.

Using the above given procedure the Laplace of probability density of the first passage time to congestion is obtained in Chapter 4. The probability density is obtained by numerically inverting the Laplace of the probability density function. If  $\pi(0) = [\pi_0(0) \dots \pi_N(0)]$  is the initial probability vector giving the probability vector for the modulating chain being in state  $i$  at time  $t=0$ . Then the probability density for the first passage time to underload state is given by

$$p_x(t) = \pi(0)p(x, t).e \quad (3.50)$$

where  $e$  is the unit column matrix.

In Section 3.3 and 3.4, expressions for probability density function of first passage time to congestion and first passage time from the congested state to uncongested state have been obtained. Next the model developed here is applied to a multiplexer that alternates switches between the congested and uncongested state, to obtain the sojourn times in these states.

### 3.5 Sojourn Times

As discussed earlier, the multiplexer switches between two states. In one state the multiplexer is considered to be in uncongested state and other state is overload or congested state. From the model of the first passage time developed here, it is observed that the sojourn period into the two states depend on the state of the modulating process  $M(t)$  and the contents of the fluid buffer  $X(t)$  at time  $t=0$ . Thus it is independent of the past states of the statistical multiplexer and the duration spent in the previous state.

It is assumed that the duration of a connection constitutes of number of On and Off periods. Within the duration of a connection, the fluid multiplexer alternates between the overload and underload states. This is a valid assumption for a multiplexer operating under a heavy load situation. The focus of this study is on better utilization of multiplexer capacity.

Let  $\{T_1, T_2, \dots, T_n, \dots\}$  be the random instants when the fluid multiplexer changes its state. In this discussion  $t = T_n$  is the instant just before the multiplexer changes state. Let  $M(T_n)$  give the state of the modulating process at time  $t = T_n$ , and  $U(T_n)$  define the state of the fluid multiplexer at the instant of interest. We define a discrete-time

process  $(M(T_n), U(T_n))$ . The process  $(M(T_n), U(T_n))$  only depends on the state of the process at the epochs of the jump and the sojourn time spent in the previous state. Hence the process  $(M(T_n), U(T_n))$ , can be described by a Semi-Markov Chain. It is assumed that the number of connections remain constant within the duration of interest.

In this section we obtain the steady state probability vector  $\pi_c = [\pi_{c0}, \pi_{c1}, \dots, \pi_{cN}]$  and  $\pi_u = [\pi_{u0}, \pi_{u1}, \dots, \pi_{uN}]$  for the modulating Markov Process  $M(t)$  at the instants when the multiplexer enters the state of congestion and uncongested state respectively. If  $X_L$  is the low threshold and  $X_H$  is the high threshold, then at the instant when the multiplexer goes into congestion, the buffer contents have just crossed the high threshold  $X_H$ . Hence the probability density for the congestion duration is given by  $p_c(X_H, t)$ . Here  $p_c(x, t)$  is the  $(N+1) \times (N+1)$  matrix for the duration of congestion given by the Laplace inversion of Equation 3.46. Let  $P_c$  be the  $(N+1) \times (N+1)$  matrix giving one step transition probability matrix for the process  $(M(T_n), U(T_n))$ .

$$P_c = \int_0^{\infty} p_c(X_H, t) dt \quad (3.51)$$

Similarly, let  $P_u$  be the  $(N+1) \times (N+1)$  matrix giving one step transition probability for the multiplexer in the uncongested state.  $P_u$  is given by

$$P_u = \int_0^{\infty} p_u(X_L, t) dt \quad (3.52)$$

where  $p_u(x, t)$  is the first passage time to congestion as defined by  $p(x, t)$  in Section 3.2. The two equilibrium probability vectors  $\pi_c$  and  $\pi_u$  satisfy the following relationships

$$\pi_c = \pi_c P_c P_u \quad (3.53)$$

and

$$\pi_u = \pi_u P_u P_c \quad (3.54)$$

The moment of the sojourn time in the two states of the multiplexer is obtained in Section 4.2 from the Laplace transforms of the probability density function for the sojourn time in congested and uncongested state. Let  $f_c(t)$  be probability density function for the congestion duration. Then  $f_c(t)$  is given by

$$f_c(t) = \pi_c p_c(X_H, t) \quad (3.55)$$

and it's Laplace is obtained by using the Laplace of the probability density time for the congestion duration as obtained in Section 3.3

$$\tilde{F}_c(s) = \int_0^{\infty} f_c(t) e^{-st} dt \quad s > 0 \quad (3.56)$$

$$\tilde{F}_c(s) = \pi_c \cdot \tilde{p}_c(X_H, s) \quad (3.57)$$

where  $\tilde{p}_c(x, s)$  is the Laplace transform with respect to the variable  $t$ , of the first passage time for the contents of the fluid buffer to fall below the low threshold  $X_L$ .

Similarly the density function for the duration of uncongested state is obtained as follows

$$f_u(t) = \pi_u p_u(X_L, t) \quad (3.58)$$

The moment of the duration of uncongested state are computed in Chapter 4 using the Laplace Transform of the probability density function as obtained in Section 3.4

## 3.6 Summary

In this chapter, we have studied in detail the model of Fluid Buffer through the sojourn time analysis. The backward Chapman-Kolmogrov equation has been used to model the first passage problem. The resulting expressions provide the necessary description of the dynamics of overload and underload duration in a statistical multiplexer. A solution for the probability density function for overload and underload periods of the multiplexer have been obtained in Laplace domain in terms of constant coefficients. These constant coefficients are solved in Chapter 4 using boundary conditions and properties of eigenvalues defined in this Chapter.

# Chapter 4

## Sojourn Time Computation

The model for obtaining the sojourn time density of the Fluid Flow Model for ATM Multiplexer using the backward Chapman-Kolmogrov equation has been discussed in the previous chapter. The solution obtained needs to be solved for the constant coefficients in Equation 3.16 and Equation 3.46 using the boundary conditions given in the Section 3.3 and Section 3.4. The explicit expressions for these constant coefficients have been obtained in terms of eigenvalues and eigenvectors of the key matrix  $D^{-1}(sI - M)$ . The expressions thereby derived are used to obtain the moments of the overload and underload periods for a simple two-state source. Comparing with those obtained from simulation validates these results. This chapter describes the procedure used for computing the results by applying the expressions derived in the previous Chapter. The Chapter also discusses the simulation approach used to validate these results.

The solution for the overload period density distribution has been obtained in Section 4.1 by solving for the constant matrix  $A(s)$  in Equation 3.46 using the boundary conditions given in Section 3.4. It also uses the boundary conditions in Section 3.3.2 to obtain the density function for underload period. In Section 4.2 the expressions for overload and underload periods are obtained for a generic two state MMRP. Such a two-state MMRP is used to approximate the superposed traffic from N On-Off sources [BAI91] [HEF86]. The numerical results in this thesis have been validated using simulator designed for the fluid flow model. The simulator is discussed in Section 4.3 and the Numerical and Simulation results are given in 4.4.

### 4.1 Constant Matrix in Sojourn Time Solution

In this Section, the solution for the probability density of sojourn times into overload and underload duration has been obtained. The probability density for first passage time to congestion has been given in Equation 3.16 by

$$\tilde{p}(x, s) = V(s)e^{z(s)x} A(s) - V(s)e^{z(s)(x-X_H)} V^{-1}(s) D^{-1} L_H \quad (4.1)$$

### 4.1.1 First Passage Time to Congestion

As discussed earlier, the eigenvalue matrix  $z(s)$  has  $\lfloor C/R \rfloor + 1$  negative eigenvalues and  $N - \lfloor C/R \rfloor$  positive eigenvalues. The constant matrix  $A(s)$  in Equation 4.1 is now obtained by applying the boundary conditions. Segregating the eigenvalues into negative and positive eigenvalues, the negative eigenvalues are indexed from 0 to  $\lfloor C/R \rfloor$ . The positive eigenvalues are indexed from  $\lfloor C/R \rfloor + 1$  to  $N$ . Let  $B(s)$  be a constant matrix given by

$$B(s) = V^{-1}(s)D^{-1}L_H \quad (4.2)$$

Let the matrices  $\tilde{p}(x,s)$ ,  $z(s)$ ,  $V(s)$ ,  $A(s)$  and  $B(s)$  be partitioned into four around the index  $\lfloor C/R \rfloor$ .

$$\tilde{p}(x,s) = \begin{bmatrix} P_1(x,s) & P_2(x,s) \\ P_3(x,s) & P_4(x,s) \end{bmatrix} \quad (4.3)$$

where  $P_1(x,s)$ ,  $P_2(x,s)$ ,  $P_3(x,s)$  and  $P_4(x,s)$  are as given below:

$$P_1(x,s) = \begin{bmatrix} p_{00}(x,s) & .. & .. & .. & p_{0,\lfloor C/R \rfloor}(x,s) \\ p_{1,0}(x,s) & .. & .. & .. & p_{1,\lfloor C/R \rfloor}(x,s) \\ .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. \\ p_{\lfloor C/R \rfloor,0}(x,s) & .. & .. & .. & p_{\lfloor C/R \rfloor,\lfloor C/R \rfloor}(x,s) \end{bmatrix} \quad (4.4)$$

$$P_2(x,s) = \begin{bmatrix} p_{0,\lfloor C/R \rfloor+1}(x,s) & .. & .. & .. & p_{0,N}(x,s) \\ p_{1,\lfloor C/R \rfloor+1}(x,s) & .. & .. & .. & p_{1,N}(x,s) \\ .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. \\ p_{\lfloor C/R \rfloor,\lfloor C/R \rfloor+1}(x,s) & .. & .. & .. & p_{\lfloor C/R \rfloor,N}(x,s) \end{bmatrix} \quad (4.5)$$

$$P_3(x,s) = \begin{bmatrix} p_{\lfloor C/R \rfloor+1,0}(x,s) & .. & .. & .. & p_{\lfloor C/R \rfloor+1,\lfloor C/R \rfloor}(x,s) \\ p_{\lfloor C/R \rfloor+2,0}(x,s) & .. & .. & .. & p_{\lfloor C/R \rfloor+2,\lfloor C/R \rfloor}(x,s) \\ .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. \\ p_{N,0}(x,s) & .. & .. & .. & p_{N,\lfloor C/R \rfloor}(x,s) \end{bmatrix} \quad (4.6)$$

$$P_4(x, s) = \begin{bmatrix} p_{\lfloor C/R \rfloor + 1, \lfloor C/R \rfloor + 1}(x, s) & .. & .. & .. & p_{\lfloor C/R \rfloor + 1, N}(x, s) \\ p_{\lfloor C/R \rfloor + 2, \lfloor C/R \rfloor + 1}(x, s) & .. & .. & .. & p_{\lfloor C/R \rfloor + 2, N}(x, s) \\ .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. \\ p_{N, \lfloor C/R \rfloor + 1}(x, s) & .. & .. & .. & p_{N, N}(x, s) \end{bmatrix} \quad (4.7)$$

Similarly, other matrices are also partitioned and are defined as

$$e^{z(s)x} = \begin{bmatrix} e^{z_A x} & 0 \\ 0 & e^{z_B x} \end{bmatrix} \quad (4.8)$$

Here  $z_A$  is diagonal matrix containing negative eigenvalues and  $z_B$  consists of all the positive eigenvalues.

$$V(s) = \begin{bmatrix} V_1(s) & V_2(s) \\ V_3(s) & V_4(s) \end{bmatrix} \quad (4.9)$$

$$A(s) = \begin{bmatrix} A_1(s) & A_2(s) \\ A_3(s) & A_4(s) \end{bmatrix} \quad (4.10)$$

$$B(s) = \begin{bmatrix} B_1(s) & B_2(s) \\ B_3(s) & B_4(s) \end{bmatrix} \quad (4.11)$$

From the assumption of linear independence of eigenvectors and the fact that the eigenvalues are distinct, the matrices  $V(s)$ ,  $W(s)$  and  $z(s)$  are all non-singular and satisfy the following relationship

$$W(s)V(s) = I \quad (4.12)$$

Hence  $B(s)$  is given by

$$B(s) = \begin{bmatrix} w_{00}(s) & .. & .. & w_{0N}(s) \\ .. & .. & .. & .. \\ ... & .. & .. & .. \\ w_{N0}(s) & .. & .. & w_{NN}(s) \end{bmatrix} \cdot \begin{bmatrix} d_{00}^{-1} & 0 & 0 & 0 \\ 0 & d_{11}^{-1} & 0 & 0 \\ .. & .. & .. & .. \\ 0 & 0 & 0 & d_{NN}^{-1} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ .. & .. & 1 & .. \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4.13)$$

Using the above definitions the matrix  $B_1(s)$ ,  $B_2(s)$ ,  $B_3(s)$  and  $B_4(s)$  are written as

$$B_1(s) = B_3(s) = 0 \quad (4.14)$$

$$B_2(s) = \begin{bmatrix} w_{0,\lfloor C/R \rfloor+1}/d_{\lfloor C/R \rfloor+1,\lfloor C/R \rfloor+1} & .. & w_{0,N}/d_{NN} \\ .. & .. & .. \\ w_{\lfloor C/R \rfloor,\lfloor C/R \rfloor+1}/d_{\lfloor C/R \rfloor+1,\lfloor C/R \rfloor+1} & .. & w_{\lfloor C/R \rfloor,N}/d_{NN} \end{bmatrix} \quad (4.15)$$

$$B_4(s) = \begin{bmatrix} w_{\lfloor C/R \rfloor+1,\lfloor C/R \rfloor+1}/d_{\lfloor C/R \rfloor+1,\lfloor C/R \rfloor+1} & .. & w_{\lfloor C/R \rfloor+1,N}/d_{NN} \\ .. & .. & .. \\ w_{N,\lfloor C/R \rfloor+1}/d_{\lfloor C/R \rfloor+1,\lfloor C/R \rfloor+1} & .. & w_{NN}/d_{NN} \end{bmatrix} \quad (4.16)$$

Applying the above definitions to Equation 4.1, and ignoring the dependence on  $s$  from  $V_i$ ,  $A_i$ ,  $B_i$  etc., the following expressions are obtained for  $P_1(x,s)$ ,  $P_2(x,s)$ ,  $P_3(x,s)$  and  $P_4(x,s)$ .

$$P_1(x,s) = V_1 e^{z_A(x)} A_1 + V_2 e^{z_B x} A_3 - V_1 e^{z_A(x-X_H)} B_1 - V_2 e^{z_B(x-X_H)} B_3 \quad (4.17)$$

$$P_2(x,s) = V_1 e^{z_A(x)} A_2 + V_2 e^{z_B x} A_4 - V_1 e^{z_A(x-X_H)} B_2 - V_2 e^{z_B(x-X_H)} B_4 \quad (4.18)$$

$$P_3(x,s) = V_3 e^{z_A(x)} A_1 + V_4 e^{z_B x} A_3 - V_3 e^{z_A(x-X_H)} B_1 - V_4 e^{z_B(x-X_H)} B_3 \quad (4.19)$$

$$P_4(x,s) = V_3 e^{z_A(x)} A_2 + V_4 e^{z_B x} A_4 - V_3 e^{z_A(x-X_H)} B_2 - V_4 e^{z_B(x-X_H)} B_4 \quad (4.20)$$

**Boundary Conditions.** By definition of first passage time to congestion it is clear that there is no possibility of the fluid buffer crossing the high threshold if the state of the modulating source  $j$  at time  $t$  is such that the net drift is negative. Therefore  $P_1(x,s)$ , and  $P_3(x,s)$  are identically zero for all values of initial fluid buffer contents  $x \leq X_H$ . Hence applying the boundary condition of Equation 3.37 to Equation 4.17 and Equation 4.19, the constant matrix  $A_1$  and  $A_3$  are obtained as:

$$\begin{aligned} V_1 e^{z_A x} A_1 - V_1 e^{z_A(x-X_H)} B_1 &= 0 \\ \therefore A_1 &= e^{-z_A X_H} B_1 \end{aligned} \quad (4.21)$$

$$\begin{aligned} V_2 e^{z_B x} A_3 - V_2 e^{z_B(x-X_H)} B_3 &= 0 \\ \therefore A_3 &= e^{-z_B X_H} B_3 \end{aligned} \quad (4.22)$$

In order to obtain  $A_2$  and  $A_4$ , the boundary condition for  $x = X_H$  is applied. If the fluid buffer contents at time  $t=0$  is  $X_H$  and the phase of the modulating process is such that the input flow is more than the link output capacity then the fluid contents will cross the high threshold at time  $t = \Delta t$  with probability 1.

Hence

$$P_4(X_H, s) = I \quad (4.23)$$

$$V_3 e^{z_A X_H} A_2 + V_4 e^{z_B X_H} A_4 - V_3 B_2 - V_4 B_4 = I \quad (4.24)$$

Boundary condition at  $x = 0$  gives the second equation in constant matrices  $A_2$  and  $A_4$ .

$$\frac{\partial P_3(0, s)}{\partial x} = 0 \quad (4.25)$$

$$V_1 z_A A_2 + V_4 z_B A_4 - V_1 z_A e^{-z_A X_H} B_2 - V_4 z_B e^{-z_B X_H} B_4 = 0 \quad (4.26)$$

Using Equation 4.24 and Equation 4.26 the following expressions for  $A_2$  and  $A_4$  are obtained:

$$A_2 = e^{-z_A X_H} B_2 - z_A^{-1} V_1^{-1} V_2 z_B \left[ V_4 e^{z_B X_H} - V_3 e^{z_A X_H} z_A^{-1} V_1^{-1} V_2 z_B \right]^{-1} \quad (4.27)$$

$$A_4 = e^{-z_B X_H} B_4 - \left[ V_4 e^{z_B X_H} - V_3 e^{z_A X_H} z_A^{-1} V_1^{-1} V_2 z_B \right]^{-1} \quad (4.28)$$

Replacing the second term in Equation 4.28 by the following approximation

$$V_4 e^{z_B X_H} - V_3 e^{z_A X_H} z_A^{-1} V_1^{-1} V_2 z_B \approx V_4 e^{z_B X_H}$$

the matrices  $P_1(x, s)$ ,  $P_2(x, s)$ ,  $P_3(x, s)$  and  $P_4(x, s)$  are given by

$$P_1(x, s) = 0 \quad (4.29)$$

$$P_2(x, s) = V_2 e^{z_B(x - X_H)} V_4^{-1} - V_1 e^{z_A x} z_A^{-1} V_1^{-1} V_2 z_B e^{-z_B X_H} V_4^{-1} \quad (4.30)$$

$$P_3(x, s) = 0 \quad (4.31)$$

$$P_4(x, s) = V_4 e^{z_B(x - X_H)} V_4^{-1} - V_3 e^{z_A x} z_A^{-1} V_1^{-1} V_2 z_B e^{-z_B X_H} V_4^{-1} \quad (4.32)$$

If  $\pi(0) = [\pi_0(0) \ . \ . \ . \ \pi_N(0)]$  is the initial probability vector giving the probability that the modulating chain was in state  $i$  at time  $t=0$ , is given, then the Laplace transform of the probability density for the first passage time to congestion is given by

$$\tilde{p}_x(s) = \pi(0) \tilde{p}(x,s).e \quad (4.33)$$

The underload state begins when the fluid buffer contents are at level  $x = X_L$ .

$P_U$  defined in Equation 3.52 is then given by

$$P_U = \tilde{p}(X_L, 0)$$

This gives the complete solution for first passage time to congestion. The solution for first passage time in overload duration is obtained next.

#### 4.1.2 First Passage Time to Underload State

The expression for the sojourn period into congestion is given in Section 3.4 by equation (3.46)

$$\tilde{p}(x, s) = V(s)e^{z(s)x} A(s) - V(s)e^{z(s)(x-X_L)} V^{-1}(s) D^{-1} L_L \quad (4.34)$$

Once again the matrices  $\tilde{p}(x, s)$ ,  $z(s)$ ,  $V(s)$ ,  $A(s)$  and  $B(s)$  are partitioned in the same manner as defined for the first passage time to congestion. Rests of the matrices remain same as defined in Equation 4.3-Equation 4.11. The definition of  $B(s)$  changes to

$$B(s) = V^{-1}(s) D^{-1} L_L \quad (4.35)$$

As the left and right eigenvalues  $V(s)$  and  $W(s)$  satisfy Equation 4.12,  $B(s)$  is given as:

$$B(s) = \begin{bmatrix} w_{00}(s) & .. & .. & w_{0N}(s) \\ .. & .. & .. & .. \\ ... & .. & .. & .. \\ w_{N0}(s) & .. & .. & w_{NN}(s) \end{bmatrix} \begin{bmatrix} d_{00}^{-1} & 0 & 0 & 0 \\ 0 & d_{11}^{-1} & 0 & 0 \\ .. & .. & .. & .. \\ 0 & 0 & 0 & d_{NN}^{-1} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ .. & .. & 0 & .. \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (4.36)$$

Using the above definitions the matrix  $B_1(s)$ ,  $B_2(s)$ ,  $B_3(s)$  and  $B_4(s)$  are written as

$$B_2(s) = B_4(s) = 0 \quad (4.37)$$

$$B_1(s) = \begin{bmatrix} w_{0,0}/d_{00} & .. & w_{0,\lfloor C/R \rfloor}/d_{\lfloor C/R \rfloor, \lfloor C/R \rfloor} \\ .. & .. & .. \\ w_{\lfloor C/R \rfloor, 0}/d_{00} & .. & w_{\lfloor C/R \rfloor, \lfloor C/R \rfloor}/d_{\lfloor C/R \rfloor, \lfloor C/R \rfloor} \end{bmatrix} \quad (4.38)$$

$$B_3(s) = \begin{bmatrix} w_{\lfloor C/R \rfloor + 1, 0} / d_{00} & \dots & w_{\lfloor C/R \rfloor + 1, \lfloor C/R \rfloor} / d_{\lfloor C/R \rfloor, \lfloor C/R \rfloor} \\ \dots & \dots & \dots \\ w_{N, 0} / d_{00} & \dots & w_{N, \lfloor C/R \rfloor} / d_{\lfloor C/R \rfloor, \lfloor C/R \rfloor} \end{bmatrix} \quad (4.39)$$

Applying the above definitions to Equation 4.34 gives the following equations for  $P_1(x, s)$ ,  $P_2(x, s)$ ,  $P_3(x, s)$  and  $P_4(x, s)$  that are similar to Equation 4.17- Equation 4.20.

$$P_1(x, s) = V_1 e^{z_A x} A_1 + V_2 e^{z_B x} A_3 - V_1 e^{z_A (x - X_L)} B_1 - V_2 e^{z_B (x - X_L)} B_3 \quad (4.40)$$

$$P_2(x, s) = V_1 e^{z_A x} A_2 + V_2 e^{z_B x} A_4 - V_1 e^{z_A (x - X_L)} B_2 - V_2 e^{z_B (x - X_L)} B_4 \quad (4.41)$$

$$P_3(x, s) = V_3 e^{z_A x} A_1 + V_4 e^{z_B x} A_3 - V_3 e^{z_A (x - X_L)} B_1 - V_4 e^{z_B (x - X_L)} B_3 \quad (4.42)$$

$$P_4(x, s) = V_3 e^{z_A x} A_2 + V_4 e^{z_B x} A_4 - V_3 e^{z_A (x - X_L)} B_2 - V_4 e^{z_B (x - X_L)} B_4 \quad (4.43)$$

**Boundary Conditions.** The solutions density function for first passage time to underload state for finite buffer case will be different from that for infinite buffer case. The solution is first obtained for multiplexer with infinite size fluid buffer. In the case of first passage time to underload state, the initial buffer contents are higher than the low threshold, hence the solution will only have terms corresponding to negative eigenvalues. The coefficients of the terms corresponding to positive eigenvalues will be zero. This is to satisfy the stability condition.

Hence letting the terms corresponding to eigenvalues in  $z_B$  in Equation 4.40 - Equation 4.43 go to zero, gives the following solution for  $A_3$  and  $A_4$

$$V_2 e^{z_B x} A_3 - V_2 e^{z_B (x - X_L)} B_3 = 0 \quad (4.45)$$

$$A_3 = e^{-z_B X_L} B_3 \quad (4.46)$$

Similarly

$$A_4 = e^{-z_B X_L} B_4 \quad (4.47)$$

$P_2(x, s)$ , and  $P_4(x, s)$  are zero as the buffer contents will not fall if the state of the modulating process at time  $t$  results in inflow being greater than the link capacity.

$$V_1 e^{z_A x} A_2 - V_1 e^{z_A (x - X_L)} B_2 = 0 \quad (4.48)$$

$$A_2 = e^{-z_A X_L} B_2 \quad (4.49)$$

$A_1$  is obtained by applying the boundary condition at  $x = X_L$ .  $P_1(x, s)$  and  $P_3(x, s)$ , corresponds to the state of the modulating process at time  $t$  being such that inflow is

less than the outflow. This if at time  $t=0$  the buffer contents are at low threshold level than

$$P_1(X_L, s) = I \quad (4.50)$$

Using Equation 4.50 in Equation 4.40  $A_1$  is obtained as

$$A_1 = e^{-z_A X_L} V_1^{-1} [I + V_1 B_1] \quad (4.51)$$

Thus the matrices  $P_1(x, s)$ ,  $P_2(x, s)$ ,  $P_3(x, s)$  and  $P_4(x, s)$  are given by

$$P_1(x, s) = V_1 e^{z_A (x - X_L)} V_1^{-1} \quad (4.52)$$

$$P_2(x, s) = 0 \quad (4.53)$$

$$P_3(x, s) = V_3 e^{z_A (x - X_L)} V_1^{-1} \quad (4.54)$$

$$P_4(x, s) = 0 \quad (4.55)$$

For multiplexer with finite fluid buffer of size  $X_B$ , the boundary conditions given in Section 3.4 are applied to obtain the density for first passage time to underload state. The constant coefficients  $A_2$  and  $A_4$  are obtained by applying the boundary condition that the fluid multiplexer cannot come out of congestion at the instant when the input flow is higher than the output flow. Applying the boundary condition of Equation 3.48 gives the following expressions for  $A_2$  and  $A_4$ .

$$V_1 e^{z_A x} A_2 - V_1 e^{z_A (x - X_L)} B_2 = 0$$

$$V_2 e^{z_B x} A_4 - V_2 e^{z_B (x - X_L)} B_4 = 0$$

$$A_2 = e^{-z_A X_L} B_2 \quad (4.56)$$

$$A_4 = e^{-z_B X_L} B_4 \quad (4.57)$$

The constant matrix  $A_1$  and  $A_3$  are obtained by applying the boundary conditions at two boundaries  $x = X_L$  and  $x = X_B$ . The boundary condition at  $x = X_L$  satisfies Equation 4.50. This leads to

$$V_1 e^{z_A X_L} A_1 + V_2 e^{z_B X_L} A_3 - V_1 B_1 - V_2 B_3 = I \quad (4.58)$$

and

$$\begin{aligned} V_3 z_A e^{z_A X_B} A_1 + V_4 z_B e^{z_B X_B} A_3 - V_3 z_A e^{z_A (X_B - X_L)} B_1 \\ - V_4 z_B e^{z_B (X_B - X_L)} B_3 = 0 \end{aligned} \quad (4.59)$$

$$A_1 = \left[ V_1 e^{z_A X_L} - V_2 e^{-z_B (X_B - X_L)} z_B^{-1} V_4^{-1} V_3 z_A e^{z_A X_B} \right]^{-1} + e^{-z_A X_L} B_1 \quad (4.60)$$

$$A_3 = e^{-z_B X_L} B_3 - e^{-z_B X_B} . z_B^{-1} V_4^{-1} V_3 z_A e^{z_A X_B} .$$

$$\left[ V_1 e^{z_A X_L} - V_2 e^{-z_B (X_B - X_L)} . z_B^{-1} V_4^{-1} V_3 z_A e^{z_A X_B} \right] \quad (4.61)$$

Since

$$V_1 e^{z_A X_L} - V_2 e^{-z_B (X_B - X_L)} . z_B^{-1} V_4^{-1} V_3 z_A e^{z_A X_B} \approx V_1 e^{z_A X_L} \quad (4.62)$$

The components of the probability density matrix  $\tilde{p}(x, s)$  are given by

$$P_1(x, s) = V_1 e^{z_A (x - X_L)} V_1^{-1} - V_2 e^{z_B (x - X_B)} . z_B^{-1} V_4^{-1} V_3 z_A e^{z_A (X_B - X_L)} V_1^{-1} \quad (4.63)$$

$$P_2(x, s) = 0 \quad (4.64)$$

$$P_3(x, s) = V_3 e^{z_A (x - X_L)} V_1^{-1} - V_4 e^{z_B (x - X_B)} . z_B^{-1} V_4^{-1} V_3 z_A e^{z_A (X_B - X_L)} V_1^{-1} \quad (4.65)$$

$$P_4(x, s) = 0 \quad (4.66)$$

Equation 4.63 clearly shows the dependence of probability density function of first passage time to underload state, on the buffer size. As  $X_B$  tends to infinity, Equations 4.63-4.66 take the form of Equations 4.52-4.55.

Using the initial vector  $\pi(0) = [\pi_0(0) \dots \pi_N(0)]$  for the state of the modulating process, Laplace transform of the probability density for the first passage time to underload state is given by

$$\tilde{p}_x(s) = \pi(0) \tilde{p}(x, s) . e \quad (4.67)$$

## 4.2 Multiplexer with Two State MMRP Source

In this section the expressions for the Laplace Transform for the first passage times have been obtained for a multiplexer fed with two state source. This two-state source with two different flow rates in each state has been used to model the approximate traffic from superposition of  $N$  On-Off type sources [BAI91]. An On-Off source is a special case of this generic two-state MMRP source. In state 0, the source generates fluid at rate  $r_0$  and in state 1 at rate  $r_1$ . This source feeds a multiplexer with infinite buffer and output link capacity C. It is assumed that  $r_0 < C < r_1$ .

For such a generic two state source defined by Markov Modulated Rate Process, the modulating chain is given by the infinitesimal generator

$$M = \begin{bmatrix} -\lambda_0 & \lambda_0 \\ \mu_1 & -\mu_1 \end{bmatrix} \quad (4.68)$$

And the drift matrix is given by

$$D = \begin{bmatrix} r_0 - C & 0 \\ 0 & r_1 - C \end{bmatrix} \quad (4.69)$$

The probability density function for the first passage time to reach the state of congestion is given by Equation 4.34 and may be written as

$$\tilde{p}_u(x, s) = \begin{bmatrix} v_{00}e^{z_0x}a_{00} + v_{01}e^{z_1x}a_{10} & v_{00}e^{z_0x}a_{01} + v_{01}e^{z_1x}a_{11} \\ v_{10}e^{z_0x}a_{00} + v_{11}e^{z_1x}a_{10} & v_{10}e^{z_0x}a_{01} + v_{11}e^{z_1x}a_{11} \end{bmatrix} - \begin{bmatrix} 0 & \frac{v_{00}e^{z_0(x-X_H)}w_{01} + v_{01}e^{z_1(x-X_H)}w_{11}}{r_1 - C} \\ 0 & \frac{v_{10}e^{z_0(x-X_H)}w_{01} + v_{11}e^{z_1(x-X_H)}w_{11}}{r_1 - C} \end{bmatrix} \quad (4.70)$$

where

$$z_0 = \frac{s(r_1 - r_0) + \lambda(r_1 - C) - \mu(r_0 - C)}{2(r_0 - C)(r_1 - C)} - \frac{\sqrt{(s(r_1 - r_0) + \lambda(r_1 - C) - \mu(r_0 - C))^2 - 4s(s + \mu + \lambda)(r_0 - C)(r_1 - C)}}{2(r_0 - C)(r_1 - C)} \quad (4.71)$$

$$z_1 = \frac{s(r_1 - r_0) + \lambda(r_1 - C) - \mu(r_0 - C)}{2(r_0 - C)(r_1 - C)} + \frac{\sqrt{(s(r_1 - r_0) + \lambda(r_1 - C) - \mu(r_0 - C))^2 - 4s(s + \mu + \lambda)(r_0 - C)(r_1 - C)}}{2(r_0 - C)(r_1 - C)} \quad (4.72)$$

Here  $z_0$  is the negative eigenvalue and  $z_1$  is the positive eigenvalue. Let there be two constants  $\alpha(s)$  and  $\beta(s)$  defined as

$$\alpha(s) = \frac{s(r_1 - r_0) + \lambda(r_1 - C) - \mu(r_0 - C)}{2(r_0 - C)(r_1 - C)} \quad (4.73)$$

$$\beta(s) = \frac{\sqrt{(s(r_1 - r_0) + \lambda(r_1 - C) - \mu(r_0 - C))^2 - 4s(s + \mu + \lambda)(r_0 - C)(r_1 - C)}}{2(r_0 - C)(r_1 - C)} \quad (4.74)$$

This gives

$$z_0 = \alpha - \beta \quad (4.75)$$

$$z_1 = \alpha + \beta \quad (4.76)$$

The right and left eigenvectors  $V(s)$  and  $W(s)$  are given by

$$V = \begin{bmatrix} 1 \\ \left[ \frac{s+\lambda}{r_0-C} - (\alpha - \beta) \right] \frac{(r_0-C)}{\lambda} \end{bmatrix} \quad W = \begin{bmatrix} 1 \\ \left[ \frac{s+\lambda}{r_0-C} - (\alpha + \beta) \right] \frac{(r_0-C)}{\lambda} \end{bmatrix} \quad (4.77)$$

$$W = \frac{1}{2\beta} \begin{bmatrix} \left[ \frac{s+\lambda}{r_0-C} - (\alpha + \beta) \right] & -I \\ -\left[ \frac{s+\lambda}{r_0-C} - (\alpha - \beta) \right] & I \end{bmatrix} \quad (4.78)$$

$$B = \begin{bmatrix} 0 & -1/2\beta(r_1 - C) \\ 0 & 1/2\beta(r_1 - C) \end{bmatrix}$$

Applying the boundary conditions described in Section 4.1, the constants are obtained using Equations 4.21, 4.22, 4.27, 4.28 as follows:

$$a_{00} = 0 \quad (4.79)$$

$$a_{01} = e^{-z_0 X_H} w_{01} / (r_1 - C) - v_{01} z_1 e^{-z_1 X_H} / (z_0 v_{00} v_{11}) \quad (4.80)$$

$$a_{10} = 0 \quad (4.81)$$

$$a_{11} = e^{-z_1 X_H} \frac{w_{11}}{r_1 - C} - e^{-z_1 X_H} \frac{1}{v_{11}} \quad (4.82)$$

This gives

Multiplexer enters the unloaded state when the fluid buffer contents fall below the low threshold. Thus at time  $t=0$ , the fluid buffer contents are at low threshold level  $x = X_L$  and the state of the Markov Process is state 0. Thus the Laplace of the probability density for unloaded period is given as

$$\tilde{p}_u(s) = -\frac{\nu_{01}z_1}{\nu_{11}z_0} e^{-z_1 X_H + z_0 X_L} - \frac{\nu_{01}}{\nu_{11}} e^{-z_1(X_H - X_L)} \quad (4.83)$$

Similarly the density distribution for congestion duration is obtained.

$$\begin{aligned} \tilde{p}_c(x, s) &= \begin{bmatrix} \nu_{00}e^{z_0x}a_{00} + \nu_{01}e^{z_1x}a_{10} & \nu_{00}e^{z_0x}a_{01} + \nu_{01}e^{z_1x}a_{11} \\ \nu_{10}e^{z_0x}a_{00} + \nu_{11}e^{z_1x}a_{10} & \nu_{10}e^{z_0x}a_{01} + \nu_{11}e^{z_1x}a_{11} \end{bmatrix} \\ &\quad - \begin{bmatrix} \frac{\nu_{00}e^{z_0(x-X_L)}w_{00} + \nu_{01}e^{z_1(x-X_L)}w_{10}}{r_0 - C} & 0 \\ \frac{\nu_{10}e^{z_0(x-X_L)}w_{00} + \nu_{11}e^{z_1(x-X_L)}w_{10}}{r_0 - C} & 0 \end{bmatrix} \end{aligned} \quad (4.84)$$

Applying the boundary conditions for the first passage time to underload state as given in Section 4.1.2, the constant coefficients are obtained using Equations 4.46, 4.47, 4.49, and 4.51, as follows:

$$a_{00} = e^{-z_0 X_L} \left[ \frac{1}{\nu_{00}} + \frac{w_{00}}{r_0 - C} \right] \quad (4.85)$$

$$a_{01} = 0 \quad (4.86)$$

$$a_{10} = e^{-z_1 X_L} w_{10} / (r_0 - C) \quad (4.87)$$

$$a_{11} = 0 \quad (4.88)$$

The initial state of the modulating process at the instant when multiplexer entered the congestion, was 1 and the contents of the fluid buffer were at level  $x = X_H$ . Thus the Laplace of the probability density for overload period is given as

$$\tilde{p}_c(s) = \frac{\nu_{10}}{\nu_{00}} e^{z_0(X_H - X_L)} \quad (4.89)$$

Similarly the Laplace transform of probability density for the congestion duration for the multiplexer with finite size fluid buffer can be obtained using Equations 4.63-4.66. The moments of congestion duration and underload period are obtained by taking the derivative of their probability density function with respect to Laplace parameter  $s$  and taking the limit  $s \rightarrow 0$ . The numerical calculations are given in Section 4.4. These are compared with the simulation results. The simulator used for this purpose is discussed next.

## 4.3 The Simulator

Discrete event simulation is a common technique for performing simulation of computer networks [AYA95] [EDW92] [AHL95] [DAL91]. It concerns the modeling of a system as it evolves over time, by a representation in which the state variables change instantaneously at separate points in time. These points are the ones at which events occur. An event is defined as an instantaneous occurrence that may change the state of the system.

In queuing system, the arrival and departure of customers or packets are the events of interest. In the realm of B-ISDN technology, there is a need to model systems with very large packet/cell rates. Most of the real time sources can be modeled as bursty sources, constant rate sources, or variable rate sources [FRO94]. The important aspect here is that the rate remains constant for some duration until the source changes state.

A simulator based on purely discrete even simulation will use arrival and departures of cells/packets as events of interest. Large numbers of cells/packets are transmitted in each burst. This requires long simulation time to get meaningful estimates of variables of concern. Simulations using the instants of change of state of the source, as event of interest, will lose the cell level statistics.

The Fast Simulator [MUD97] described here uses discrete simulation and analysis where discrete events are instants when change in traffic occurs. In between the discrete events, the continuous variable like queue-size is governed by differential equations based on flow model. Such a simulator will require less memory space as an actual queue does not have to be kept in the form of linked list, as would be done in the case of discrete even simulator. Queue size for this simulator will be just a variable.

The Fast Simulator had been developed keeping in mind the simulation of ATM networks based on virtual circuits and virtual paths. With some additional programming, the simulator can be used for other classes of telecommunication networks.

### 4.3.1 Simulation Technique

Simulation technique used in Fast Simulator is a combination of discrete event simulation and flow analysis. State variables are divided into discretely changing and continuously changing category. Three fundamental types of interactions occur

between discretely changing and continuously changing state variables [PRI86] [Law91]

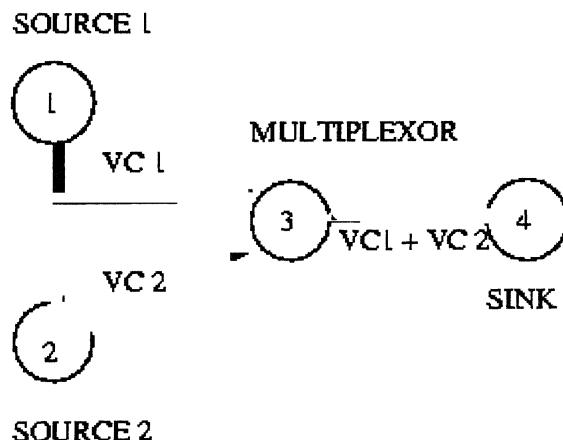
- A discrete event may cause a discrete change in the value of a continuous state variable
- A discrete event may cause the relationship governing a continuous state variable to change at a particular time.
- A continuous state variable achieving a threshold value may cause a discrete event to occur or to be scheduled.

In the proposed Fast Simulation technique, queue-size at a node is a continuous state variable. The following discrete events result in the change of state in a node leading to change in dynamics governing the continuous state variable queue size.

- State change of a traffic source
- Change in the input traffic rate at a queue
- Change in the output traffic rate at a node
- Queue size reaching a threshold or a boundary
- Control signal sent by one node to another etc.

#### 4.3.2 Basic Model

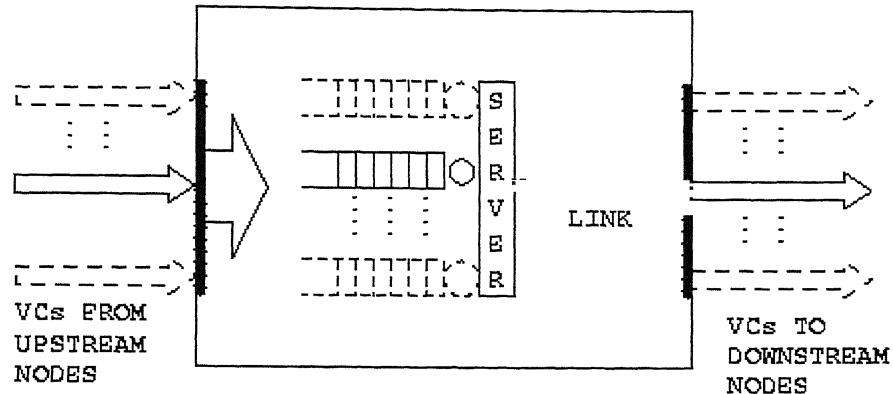
An ATM network can be modeled as interconnection of switches and multiplexers with virtual circuits routed through them Figure 4.1. A NODE and a TRAFFIC SOURCE are the two basic elements built into the simulator.



**Figure 4.1 An ATM Multiplexer Model**

NODE: In this simulator basic element is a NODE with one or more queues, served by a single unidirectional link of Capacity C. Node acts like a multiplexer

switch, multiplexing the incoming virtual circuits and switching them to respective output ports for down stream nodes Figure 4.2. A Virtual Circuit starts from a SOURCE node and ends at a SINK node passing through one or more intermediate nodes.



**Figure 4.2 Simulator Node Element**

Discrete events for each node are changes in input rate on a virtual circuit to the node and control signal to the node. Queue-size is a continuous state variable whose rate of change is described by the differential equation:

$$\frac{d}{dt} \text{queue\_size} = R_{in} - R_{out} \quad (4.99)$$

where  $R_{in}$  and  $R_{out}$  of a queue are aggregate input and output rates respectively.

The events concerned with queue-size reaching a threshold is what [PRI86] calls a state event. Unlike discrete events, state events are not scheduled but occur when a continuous state variable crosses a threshold.

**TRAFFIC SOURCE:** Virtual circuit traffic source is assumed to be a variable rate source with instants of change of rate being governed by some user defined Markovian/ NonMarkovian distribution. These instants can also be read from a data file. In the example used for comparison in Section 4.3.3, the traffic source has been described as superposition of N On-Off type sources with exponentially distributed On and Off periods.

### 4.3.3 Simulator Features

The present version of Fast simulator has been implemented in C Language in Unix/Linux environment.

## DESIGN DETAILS

The Simulator takes as input a network of nodes, virtual circuits with traffic source definitions and paths. Call Admission Control (CAC) procedure can be run on every NODE on the path of a virtual circuit before adding a virtual circuit to the network. Once a network has been defined along with virtual circuits, any node in the network can be loaded to desired factor by using AUTO\_LOAD procedure. AUTO\_LOAD adds virtual circuits on a specified node so that the node is loaded to the desired factor. This is done without disturbing the existing virtual circuits and other nodes. This can be used to obtain performance parameters under different load conditions.

As the simulation proceeds, simulation clock is advanced based on next-event time advance mechanism. The simulator manages a central list of events. This list contains a pointer to head-of-list event of each node, in the order of time of maturity. When an event corresponding to a node matures, the next event in the node's event list is inserted in the central list of events. The distributed management of event lists reduces the computer time used in inserting an event in the order of maturity of time.

When an event corresponding to a node is scheduled, it refreshes the value of continuous state variables by solving the differential equations based on fluid flow model for these variables. The initial state is taken, as that of the last event time and the time increment is the interval between the occurrence of two events for that node.

A brief description of action taken on different events is as follows:

- SOURCE\_RATE\_CHANGE event changes the traffic rate at the source node of a virtual circuit. It also creates an event for the next due instant of state change. The next due instant of state change of a source is decided based on source definition provided by user or available in library. For generating random distributions, each source is assigned one or more independent stream of random numbers.
- INPUT\_RATE\_CHANGE event at a node changes the input rates of the virtual circuits based on the information passed by the upstream nodes. It then invokes BANDWIDTH\_MANAGER to reallocate the output link bandwidth between different virtual circuits.
- OUTPUT\_RATE\_CHANGE event at a node invokes BANDWIDTH\_MANAGER to change the output rates of virtual circuits

and insert INPUT\_RATE\_CHANGE event in downstream nodes' event lists. BANDWIDTH\_MANAGER then invokes BUFFER\_MANAGER which evaluates the instant at which queue size will reach one of the user defined thresholds, and inserts a BUFFER\_CONTROL event in the node event list. This time is evaluated based on equation for queue size.

- BUFFER\_CONTROL event at a node invokes BUFFER\_MANAGER, which in turn invokes predefined procedures for predefined thresholds, or user defined procedures for thresholds defined by user.
- NODE\_CONTROL event for a node is generated by upstream or downstream nodes and is used to run a control protocol as defined by the user.

To implement different strategies for bandwidth management, buffer sharing and feedback controls, small code in C language need to be added to the existing procedures.

## STATISTICS COLLECTION

STATISTICS\_MANAGER initialises and manages the statistics needed by the user. At the end of a simulation run, the estimates are printed in file RESULTS. STATISTICS\_MANAGER collects statistics for each node, statistics of each virtual circuit at each node on it's path, and end to end statistics of virtual circuits. First and Second moments of input rate, output rate, queue size probability density, delay, loss periods etc at a node, input rate, output rate, delay, loss probability of a virtual circuit at a node and end to end parameters are saved by STATISTICS\_MANAGER, along with confidence interval based on t-distribution.

## RANDOM NUMBER GENERATOR

Random Number generator uses different seeds for each random variable thus reducing the possibility of correlation between different sequences. Continuous random variates like Uniform, Exponential, m-Erlang, Gamma, Normal etc., can be used for defining a source.

### 4.3.4 Fast Simulator Validation

For performance comparison, a single multiplexer was modeled and simulated on a simulator based on purely Discrete Event simulation (D-SIM) and the one proposed here (FAST). Two sources, each a superposition of ON-OFF sources with exponentially distributed on and off periods and peak rate R, sent traffic into a

multiplexer with buffer size of 6000 cells and Link Capacity of 1,500,000 cells/s Table 4.1.

|                              | <b>Source 1</b> | <b>Source 2</b> |
|------------------------------|-----------------|-----------------|
| <b>No. of On-Off Sources</b> | 130             | 120             |
| <b>On Period in sec.</b>     | 0.1             | 0.1             |
| <b>Off Period in sec.</b>    | 0.4             | 0.9             |
| <b>Peak Rate in cells/s</b>  | 25000           | 25000           |

**Table 4.1 Source Characteristics**

| <b>Simulation Run Time</b> | <b>Computer Time Used</b> |             | <b>Gain</b> |
|----------------------------|---------------------------|-------------|-------------|
|                            | <b>DSIM</b>               | <b>FAST</b> |             |
| 1s                         | 29s                       | 0.65s       | 45          |
| 10s                        | 331s                      | 7s          | 47          |
| 20s                        | 646s                      | 13s         | 49          |
| 50s                        | 1499s                     | 29s         | 50          |
| 100s                       | 2722s                     | 54s         | 50          |

**Table 4.2 Computer Time Used**

| <b>Variable</b>  | <b>Simulation Results</b> |             |
|------------------|---------------------------|-------------|
|                  | <b>D-SIM</b>              | <b>FAST</b> |
| Throughput       | 0.78                      | 0.78        |
| Loss Probability | 0.00044                   | 0.00044     |
| Average Delay    | 49.5 $\mu$ s              | 50 $\mu$ s  |
| Maximum Delay    | 0.004 s                   | 0.004 s     |

**Table 4.3 Simulation Results**

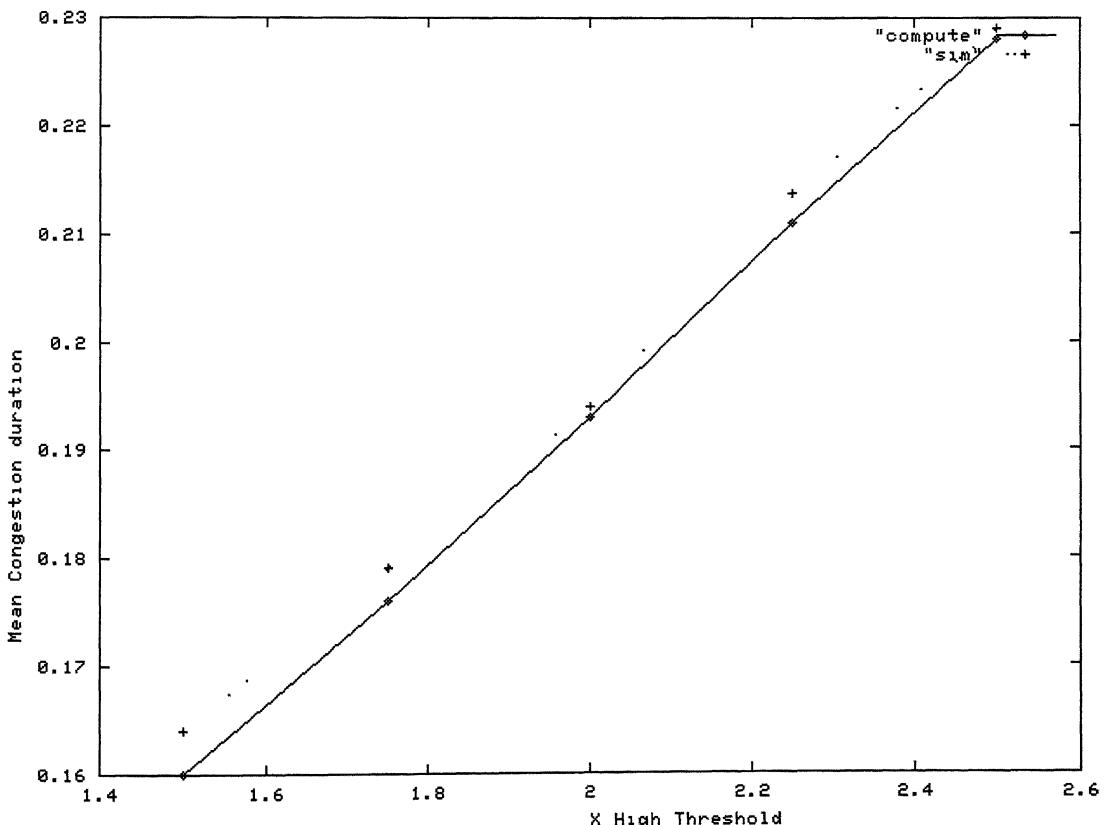
Table 4.2 gives the computer time taken for different lengths of simulation run for the two simulators. Machine used for simulation was digital DEC 3000 workstation. Some of the statistics obtained for the multiplexer node using two simulators is given in Table 4.3. Queue size probability density curve obtained from the two simulators matched. In

Table 4.3, there is a mismatch of average delay parameter. This mismatch is attributed to the discrete nature of queue size increment in Discrete simulator and continuous increase in queue size in case of Fast Simulator. By reducing the mean ON period, it was observed that performance gain reduced as the mean ON period got closer to cell transmission time.

One limitation of the Fast Simulator is that moments of end to end delay parameter cannot be obtained since tagging of individual cells is not possible. Cell to cell variation of delay jitter also cannot be obtained for the same reason. However it is adequate for most of the parameters of our interest.

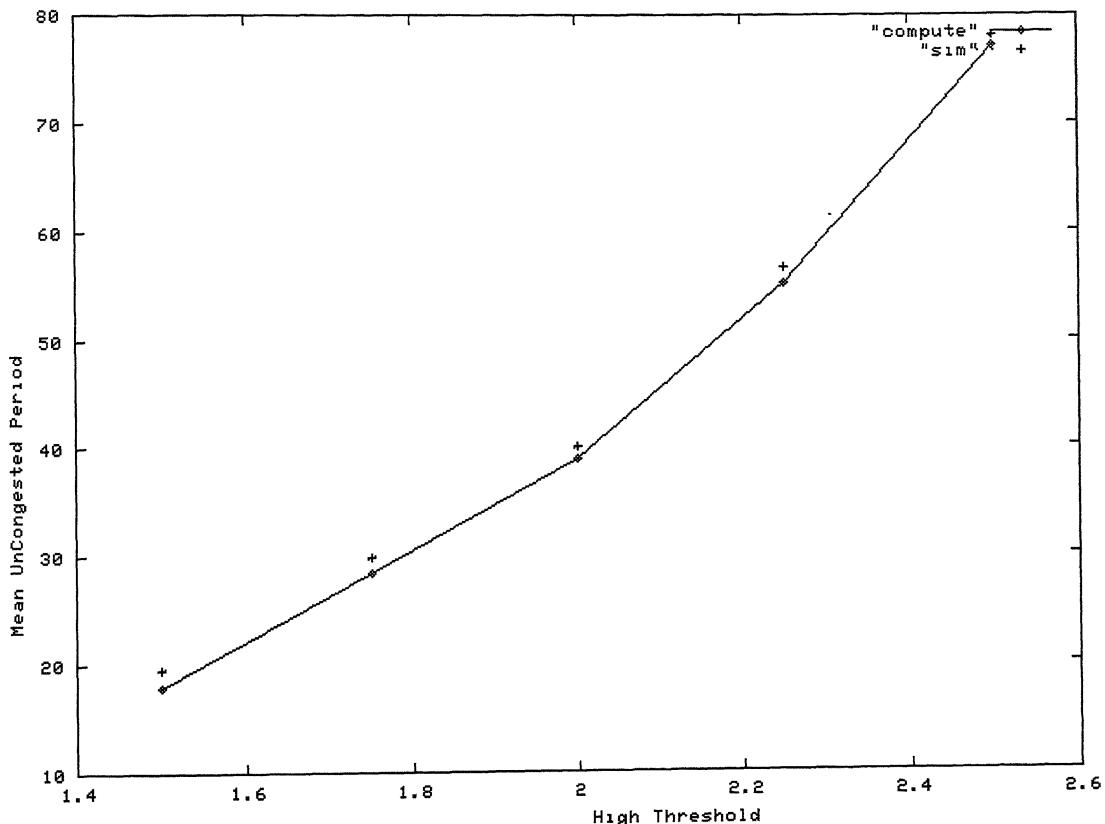
## 4.4 Numerical Results

In this section the numerical results are obtained for validating the sojourn-time model developed in Chapter 3 and Chapter 4. The mean sojourn periods in the congested and uncongested state are computed and compared with the results obtained using the fast simulator described in Section 4.3.



**Figure 4.3 Mean Congestion Duration Vs High Threshold ( $X_L$  constant)**

An example of a fluid multiplexer fed with 50 On-Off sources with exponentially distributed ON and Off periods has been considered. The mean On period is 100 ms and mean off period is 200 ms. In the On state each source generates traffic at the rate of 4 Mbps. No traffic is generated in the off state. The multiplexer is finite buffered with buffer size of 3.0 Mbits. The Numerical results illustrate how the mean congestion duration varies with the change in threshold values and the link capacity C. Link bandwidth of 85 Mbps is used for simulation and numerical results. For the computation purpose, the superposition of sources was approximated as two state MMRP. The approach used for this is described in Appendix 4.

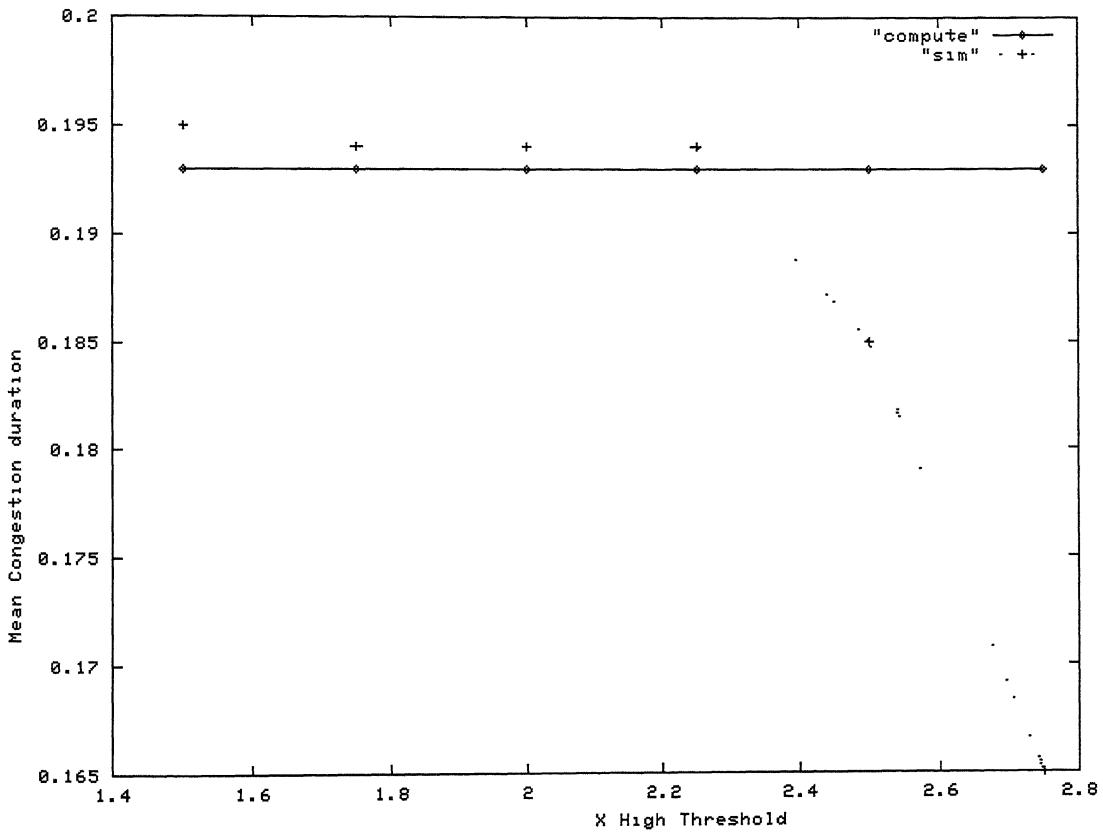


**Figure 4.4 Mean Underload Period vs High Threshold**

In first example, high threshold is varied keeping the low threshold at 1 Mbits. In Figure 4.1, mean congestion duration in sec is plotted against the high threshold in Mbits. As high threshold increases it is observed that the mean congestion duration increases linearly with the increase in high threshold. The computed results closely match with those obtained from simulation. Result show that computed mean is lower than that obtained from simulation. A possible reason for this is two-state

approximation used. Two state approximation results in more frequent rate change in the input as compared with the superposed arrival from  $N$  sources.

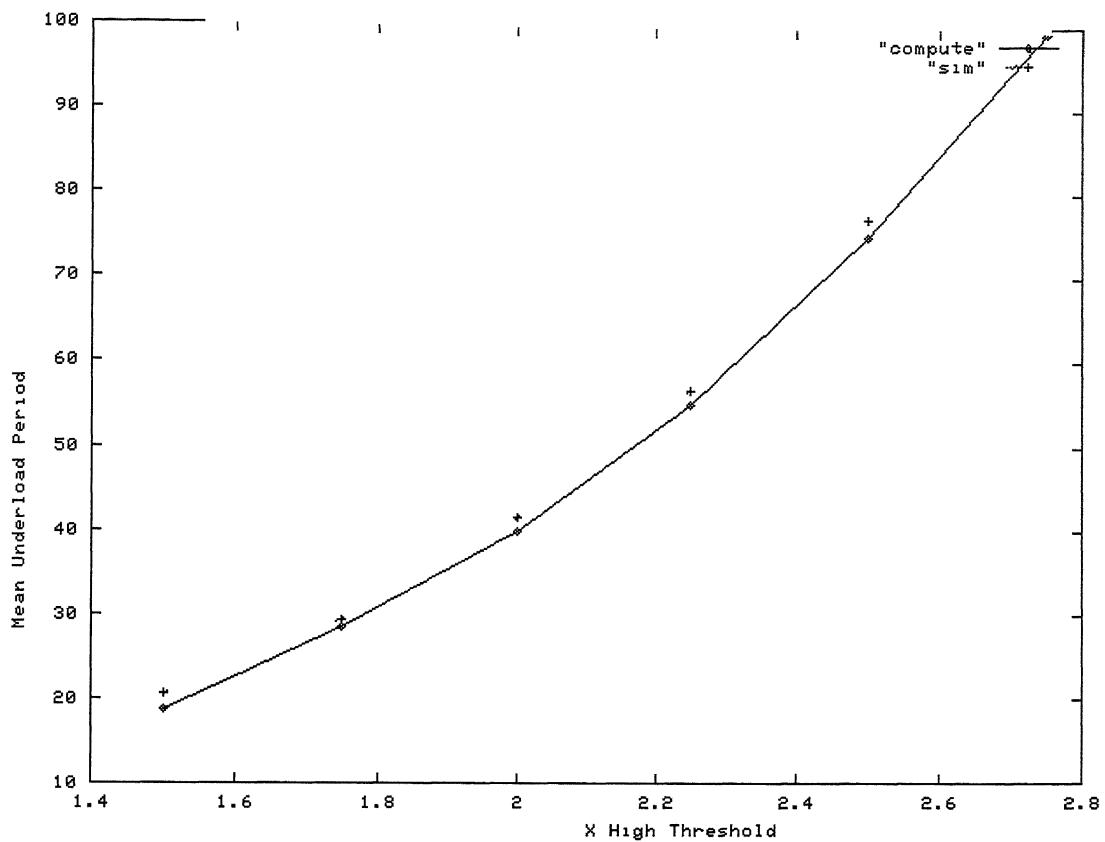
The mean underload period is plotted against high threshold in Figure 4.4. With increase in high threshold, the mean duration in underload state also increases. It is observed that the mean underload period shows exponential increase with increase in High Threshold. This is because of the terms corresponding to positive eigenvalues in the expression for probability density of sojourn time in underload state (Equation 4.83). The computed results are similar to those obtained from simulation.



**Figure 4.5 Mean Congestion Duration with Increase in High and Low Thresholds**

Next we observe the behavior of the sojourn times as both high and low thresholds are varied keeping their difference constant. The difference between high threshold and low threshold is kept constant at 1 Mbits. The plot in Figure 4.5 shows the variation in congestion duration with change in high and low thresholds keeping the difference constant. While computed result using Equation 4.89 shows that the mean congestion duration remains constant, the simulation shows a drastic fall in the mean

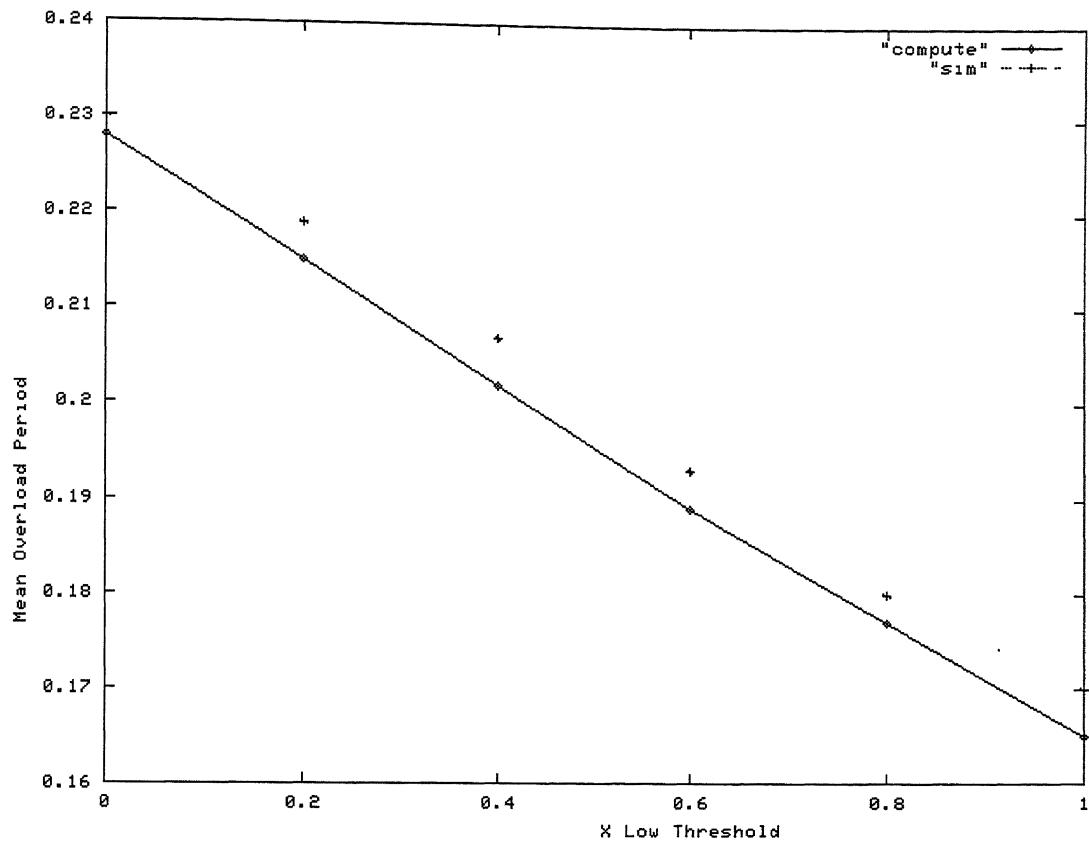
congestion duration as high threshold gets closer to the buffer capacity of the multiplexer. This is because of use of finite sized buffer (3 Mbits) in simulation.



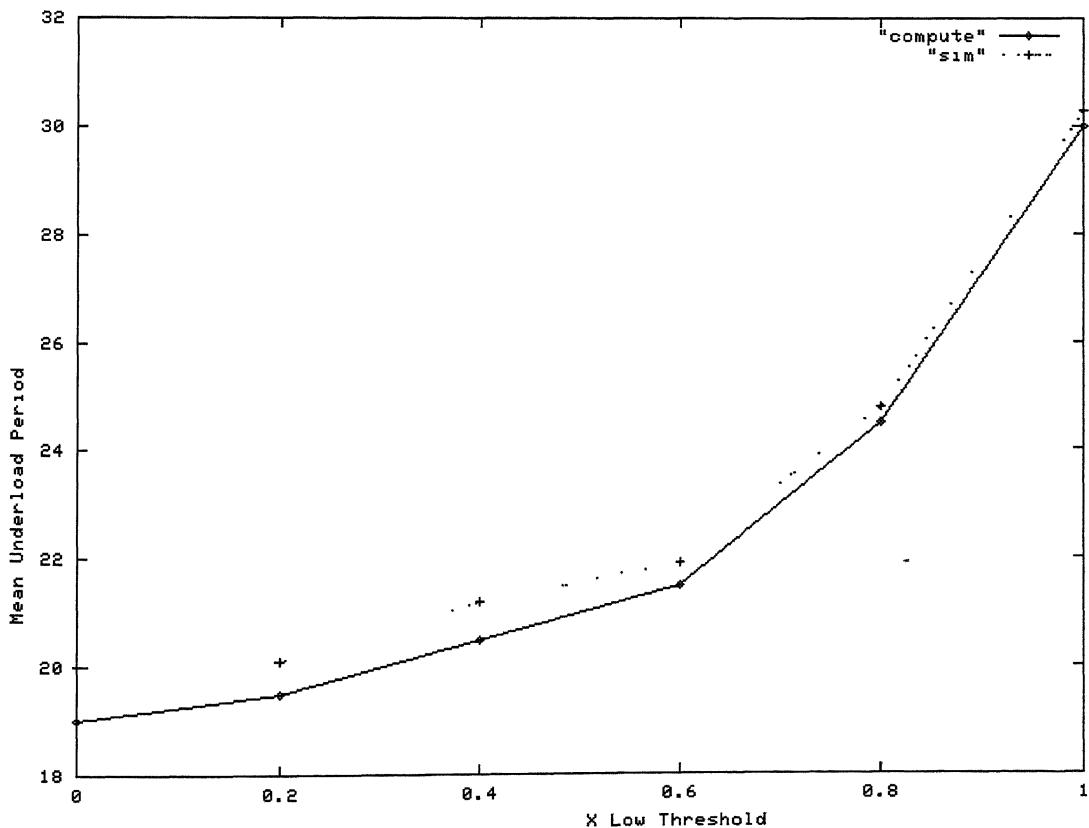
**Figure 4.6 Mean Underload Period with increase in High and Low Thresholds**

The sojourn time in uncongested state for the case where the difference between high and low threshold is maintained constant is plotted in Figure 4.6. Mean sojourn time in underload state shows exponential dependence on high and low thresholds. Comparing the two curves of Figure 4.4 and Figure 4.6 it is observed that the mean sojourn time in underload state shows slightly higher increase as both low and high thresholds are increased simultaneously in comparison where only high threshold is increased. This is an interesting observation. Otherwise the two curves of Figure 4.4 and Figure 4.6 are almost the same.

The impact of change in low threshold is shown next. The high threshold is kept fixed at 1.5 Mbits while low threshold is varied Figure 4.7 and Figure 4.8. Increase in low threshold reduces the mean congestion duration almost linearly Figure 4.7 while increasing the mean underload period in an exponential manner Figure 4.8.



**Figure 4.7 Overload Period vs Low Threshold**



**Figure 4.8 Underload Period vs Low Threshold**

## 4.5 Summary

In this chapter the procedure for computing the mean sojourn periods has been shown along with the results using the numerical computation and simulation. The simulator developed for simulations has been described comparing the results obtained from the fluid flow simulator with those obtained from discrete event simulator. Superposed traffic from  $N$  On-Off sources is approximated by two-state MMRP and the mean congestion and underload duration have been obtained to show the behaviour of sojourn time with variation in high and low threshold. The results match with those obtained from simulation. The procedure and model for the sojourn period density obtained in this chapter is used in the next chapter to model and study the behavior of sojourn periods of the multiplexer with feedback controlled sources.

## Chapter 5

# Feedback Controlled Source Model

In this Chapter performance of a statistical multiplexer with feedback controlled sources is analyzed using the approach developed in Chapter 3 and 4. The multiplexer alternates between overload and underload states. While in the model used in Chapter 3 there was no control applied to the fluid sources, in the model used in this Chapter, the input to the multiplexer is deterministically regulated when the multiplexer is in congested state. In this model when a multiplexer enters the overload state, it sends an explicit feedback to the sources to reduce their rate. The sources reduce their output to a lower rate that is predetermined based on negotiations at call setup time. This rate may be same as guaranteed bandwidth for each connection. A source can regulate its output in two ways. One is by discarding the excess cells/packets in order to maintain the lower rate. Other option is of using source buffer to smoothen the burstiness in the traffic. While in the first case the modulating process retains its original characteristic, in the second case the characteristic of the modulating process is changed. The change in flow of fluid input changes the sojourn times into overload and underload states. The model defined in this Chapter is used to study the performance and effectiveness of binary feedback in reducing the duration for which a multiplexer remains in congestion. We also study the effect of propagation delay, regulated flow rate and source buffer on the performance of feedback mechanism.

In this Chapter the congestion duration and underload periods are modeled for feedback controlled sources. Section 5.1 discusses the Model used. Section 5.2 obtains solution for the case where controlled sources do not have source buffer. Model with source buffer is dealt with in Section 5.3. The effect of non-negligible propagation delay is studied in Section 5.4. The performance is studied using Numerical Examples in Section 5.5.

### 5.1 The Model

The statistical multiplexer is modeled as a fluid buffer with constant rate output. The multiplexer is fed by N On-Off sources with exponentially distributed On and Off

periods. At any instant a multiplexer is considered in either of the two states, congested (overload state) or uncongested state (underload state). The overload state and underload state have been defined earlier in Section 3.1. On entering into congestion a multiplexer sends explicit binary feedback to the sources forcing the sources to control the source output. The binary feedback is also sent when multiplexer changes state from overload state to underload state. When a multiplexer enters the underload state, it sends feedback to all sources which results in removal of control on source output rate. This feedback allows the sources to transmit at their peak rates. This model is further extended to incorporate non-negligible feedback propagation delay. The signal from multiplexer reaches a source after propagation delay of  $d_b$ . Based on this signal, the source changes its output rate. This change of rate by the source is observed in the arrival process at the multiplexer after another propagation delay of  $d_f$ . Thus the combined round trip delay between the instance when feedback signal is generated and the instance when changes due to the signal are observed in the arrival process at the multiplexer input is  $d_t$ .

$$d_t = d_f + d_b \quad (5.1)$$

The different variations of the feedback model have different applications in the context of ATM. Three variations of this model are being analysed to observe the performance of these when various parameters are changed. The performance measures under consideration are the mean overload and underload duration. First model that is being considered in this chapter is of feedback controlled On-Off Sources with two possible rates. When multiplexer is in uncongested state, fluid sources generate traffic at peak rate in their On periods. The source output is said to be in uncontrolled state. When the multiplexer enters the state of congestion, it sends a signal to the source and source immediately lowers the traffic rate. The propagation delay is assumed to be negligible. Such a model typically fits into an adaptive traffic regulator at the input of an ATM multiplexer. Such a traffic regulator discards all the tagged cells and only allows the cells that are not tagged to pass through to the multiplexer. Leaky bucket mechanisms being a special case to which this model is applicable.

In second model, each On-Off source has an associated source buffer. When the multiplexer is in underload state, the traffic flows out of this source buffer at a peak rate R. On entering overload state, the multiplexer sends congestion feedback signal to each source. The output of the source buffer enters the controlled or regulated state on

receiving this signal. The flow from the source buffer is reduced to controlled rate  $R_1$ . Once again the propagation delay between the statistical multiplexer and the source is assumed to be negligible. This model is applicable to traffic shaper at the input of a statistical multiplexer in the ATM network. It is also applicable to the sources with traffic smoothening buffer at the edge of the network.

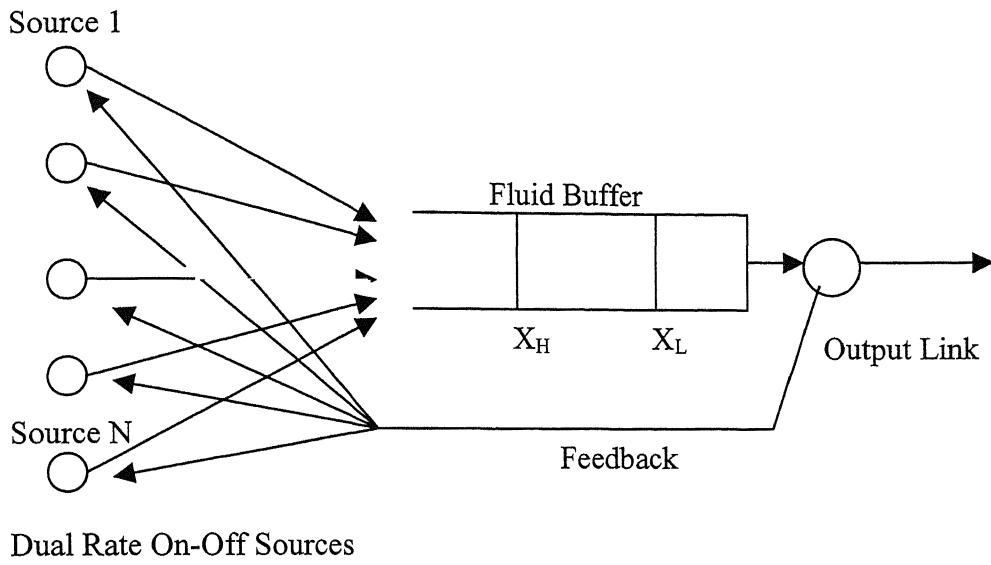
Third model is similar to the second one except that the propagation delay is no longer negligible. For the simplicity of analysis this round trip delay has been assumed to be constant for all sources. This model is applicable for modeling back-pressure mechanism with the source representing an upstream node whose output is regulated based on the state of the downstream multiplexer. Though there is a feedback mechanism which links the two nodes (source and multiplexer), this mechanism comes into the category of loosely coupled system, as there is no continuous regulation of traffic by the statistical multiplexer. Regulation only comes into play when the multiplexer under study enters into congestion. Though the first two models are only special cases of this model, they are being discussed separately so as not to miss the salient points of these models in the complexity of the more generic third model. The governing equations for the sojourn time into overload and underload states for the generic third model is derived from that of more simpler first two models.

Multiplexer with feedback controlled sources with source buffers and non-negligible feedback propagation delay would form a good model to represent the two-cascaded nodes in the network with backpressure mechanism of flow control. It only requires modeling the output of the statistical multiplexer to be varying with some kind of two state modulating process. In this work the output of the statistical multiplexer has been assumed to be deterministic with constant rate  $C$ . In all the cases the feedback mechanism is considered to be simple binary feedback.

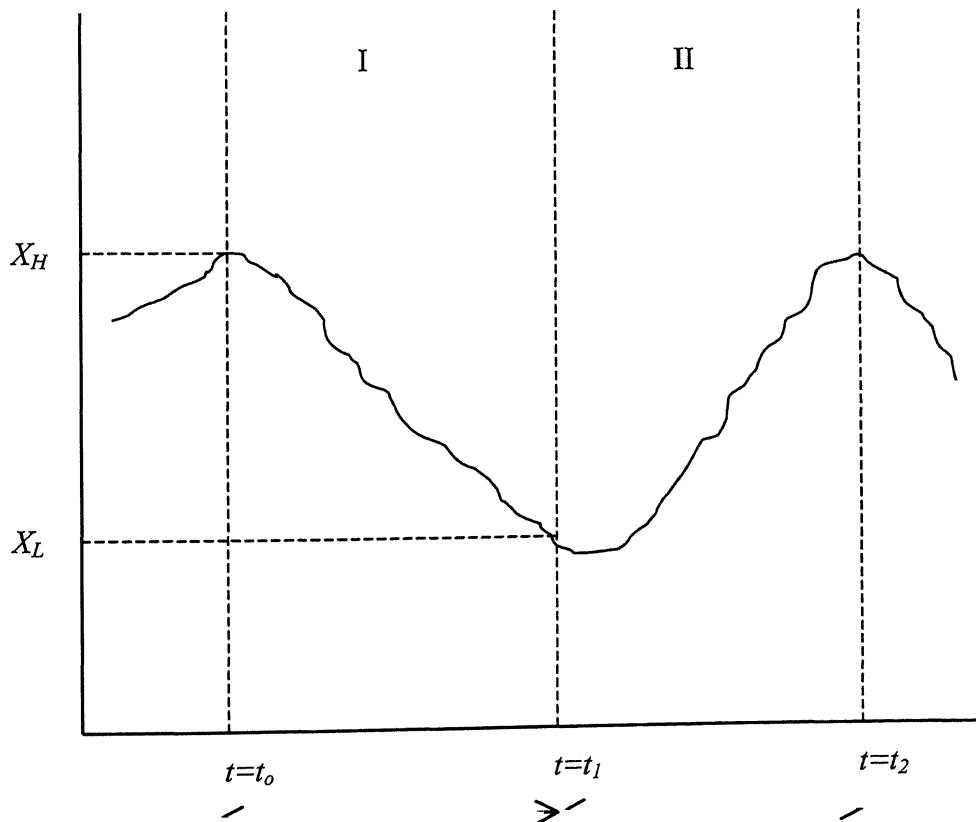
## 5.2 Multiplexer with Controlled Sources

A typical regulator at the input of an ATM switch/multiplexer is being modeled here. There is no propagation delay between the multiplexer and the regulator. The flow regulator feeds On-Off traffic to the statistical multiplexer at the peak rate  $R$  when the multiplexer is in underload state. When the multiplexer is in overload state, the input flow to the multiplexer is reduced. The regulator regulates the input flow to the multiplexer to reduced rate  $R_1$  and discards the additional fluid. There is one regulator for each Virtual Circuit or Virtual Path that passes through the ATM multiplexer. This

is being modeled by a statistical multiplexer with feedback controlled sources Figure 5.1.



**Figure 5.1 Sources with Feedback Controlled Output Rate (Model 1)**



**Figure 5.2 A Sample Path of Fluid Buffer Contents with Time (Model 1)**

The statistical multiplexer of Figure 5.1 is fed by  $N$  homogeneous On-Off sources with exponentially distributed On and Off periods. The flow from each source is regulated by the feedback from the multiplexer. At instant  $t = t_0$  (Figure 5.2) the multiplexer enters into congestion as the fluid contents cross the high threshold  $X_H$ . Region  $I$  corresponds to the overload period of the multiplexer. In this region the output of the sources is regulated at rate  $R_I$  and the excess traffic is discarded. This discard mechanism is either implemented at the traffic regulator for each Virtual Circuit at the input of a multiplexer or at the traffic source itself to provide graceful degradation by changing the output rate. Most of the sources generating multimedia traffic like Video and Audio (e.g. MPEG-4) support this kind of scalability and graceful degradation. At  $t = t_1$  the fluid buffer contents fall below low threshold  $X_L$  and enter underload period given by region  $II$  of Figure 5.2. In region  $II$  the sources are in uncontrolled mode and generate traffic at their peak rate  $R$ . The multiplexer remains in underload state till it again enters into congestion at time  $t = t_2$ . The combined fluid arrival at the input of a statistical multiplexer is once again modeled as  $N+1$  state MMRP as in Section 3.1. Feedback only changes the output rate from the source, it does not change the state of the On-Off source and hence continuity of the state of MMRP at the embedded instants  $t_0, t_1, t_2, \dots$  is assumed. The fluid buffer is assumed to be infinite.

When multiplexer is in uncongested state, the MMRP fluid input rate  $r_i$  in state  $i$  of the modulating birth and death process is given by  $r_i = iR$ . The infinitesimal generator  $M$  for the continuous time birth and death modulating process is given by

$$M \equiv \begin{bmatrix} -N\lambda & N\lambda & . & . & 0 \\ \mu & -(N-1)\lambda - \mu & (N-1)\lambda & . & 0 \\ . & . & . & . & . \\ 0 & . & . & . & \lambda \\ 0 & . & . & . & N\mu - N\mu \end{bmatrix} \quad (5.2)$$

and the drift matrix  $D$  is given by

$$D \equiv \begin{bmatrix} -C & 0 & . & . & 0 \\ 0 & R-C & 0 & . & 0 \\ . & . & . & . & . \\ 0 & . & . & . & 0 \\ 0 & 0 & 0 & 0 & NR-C \end{bmatrix} \quad (5.3)$$

where  $R$  is Peak Rate of the On-Off source,  $1/\mu$  is the mean On period,  $1/\lambda$  is the mean Off period of the On-Off source and  $C$  is the link capacity of the fluid multiplexer output. This defines the MMRP governing the input flow to the fluid buffer of Figure 5.1 in region  $II$  of Figure 5.2. In region  $I$  the mean On and Off periods of the On-Off sources remain same as in region  $II$ . The output reduction is achieved by discarding part of the traffic generated by the source and has no impact on the On-Off characteristic of the On-Off source and hence on the Continuous Time Markov Chain which defines the MMRP. The change in input flow rate in region  $I$  and  $II$  implies that the buffer dynamics and hence the equations governing the first passage times are different in two regions. If the Continuous Time Markov Chain for the modulating MMRP in regions  $II$  is given by  $M$  (Equation 5.2) then the infinitesimal generator  $M_I$  defining the input flow modulating process in region I is given by Equation 5.4. Drift  $D$  in region II is given by Equation 5.3 and drift  $D_I$  in region I is given by Equation 5.5.

$$M_I = M \quad (5.4)$$

$$D_I \equiv \begin{bmatrix} -C & 0 & . & . & 0 \\ 0 & R_I - C & 0 & . & 0 \\ . & . & . & . & . \\ 0 & . & . & . & 0 \\ 0 & 0 & 0 & 0 & NR_I - C \end{bmatrix} \quad (5.5)$$

where  $R_I$  is the controlled output from an On-Off source.

The solutions obtained in Section 4.1 for the sojourn time into overload and underload state are applied here to obtain the congestion duration and underload period density function for the model given here.

### 5.2.1 Underload Period for Model 1

The key matrix governing the probability density for the first passage time to congestion is given by  $D^{-1}(sI - M)$ . This is same as the key matrix governing the dynamics of the first passage time to congestion in Section 3.2. Hence the introduction of feedback does not have any impact on the density function for the first passage time to congestion.

The solution for Laplace of density function for first passage time to congestion as defined in Section 3.1 is given by Equation 4.29-4.32

$$\tilde{p}(x, s) = \begin{bmatrix} 0 & V_2 e^{z_B(x-X_H)} V_4^{-1} - V_1 e^{z_A x} z_A^{-1} V_1^{-1} V_2 z_B e^{-z_B X_H} V_4^{-1} \\ 0 & V_4 e^{z_B(x-X_H)} V_4^{-1} - V_3 e^{z_A x} z_A^{-1} V_1^{-1} V_2 z_B e^{-z_B X_H} V_4^{-1} \end{bmatrix} \quad (5.6)$$

where  $V_1, V_2, V_3, V_4, z_A, z_B$  are defined in Section 4.1.1 and are given by the following:

$$z(s) = \begin{bmatrix} z_A & 0 \\ 0 & z_B \end{bmatrix} \quad (5.7)$$

Here  $z_A$  consist of negative eigenvalues of the key matrix  $D^{-1}(sI - M)$  and  $z_B$  constitutes of positive eigenvalues. The solution for eigenvalues is given by (3.33) and the eigenvector  $V(s)$  is obtained using (3.34) and (3.35)

$$V(s) = \begin{bmatrix} V_1(s) & V_2(s) \\ V_3(s) & V_4(s) \end{bmatrix} \quad (5.8)$$

From the definition of the underload period itself, the initial level of fluid buffer at start of underload period is given by  $x = X_L$ . The state of the modulating process at start of underload period depends on state of the MMRP in which the previous overload period ended. Consider the process that assumes the state of the modulating process at the instances where the underload period is entered. This process is actually an embedded Markov Chain with its equilibrium probabilities defined by the row vector  $\pi_u$ . The Laplace Transform of the density function for sojourn time in underload state is given by

$$\tilde{p}_u(s) = \pi_u \tilde{p}(X_L, s).e \quad (5.9)$$

The equilibrium probability vector  $\pi_c$  for the process that assumes the state of the modulating MMRP process at the instances when the multiplexer enters the congestion state is given by

$$\pi_c = \pi_u P_u \quad (5.10)$$

where  $P_u$  is obtained using

$$P_u = \tilde{p}(X_L, 0) \quad (5.11)$$

Thus if introduction of feedback will have any effect on mean underload period then it will be because of change in equilibrium probability vector  $\pi_u$ . For a two state MMRP of Section 4.2, the equilibrium probability vector remains same as that for open loop case (Section 4.1.1) and hence will have no impact of feedback on mean soujourn time in underload state.

## 5.2.2 Overload Period for Model 1

Though the modulating process for the flow input to the multiplexer remains the same as that for underload period, the rate of input flow in each state changes thus changing the parameters of the drift  $D_1$  given by Equation 5.5. The first passage time to underload state is obtained using the solution for infinite fluid buffer case in Section 3.4 and Section 4.1.2. Since the key matrix  $D_1^{-1}(sI - M_1)$  in case of feedback controlled sources is different from that for sources with no control (Section 3.2), hence the eigenvalues and eigenvectors for the key matrix are different. This implies that adding feedback control results in change in overload behavior of the statistical multiplexer.

The overload control mechanism in general requires the peak input rate to the statistical multiplexer to be less than the output link capacity. A revisit to the properties of eigenvalue, given in Appendix 1, gives that there are  $N+1$  negative eigenvalues if the regulated rate  $R_1$  is such that  $NR_1 < C$ .

If  $z(s)$  is the eigenvalue matrix for the key matrix  $D_1^{-1}(sI - M_1)$  and  $V(s)$  the right eigenvector and  $W(s)$  is the left eigenvector then, the solution for Laplace of density function for first passage time to underload state is obtained using Equation 4.52

$$\tilde{p}(x, s) = V e^{z(x-X_L)} W \quad (5.12)$$

Multiplexer enters the state of congestion with the fluid buffer contents at level  $x=X_H$ . The equilibrium probability vector for the state of the modulating process at the beginning of congestion duration,  $\pi_c$ , is given by Equation 5.10. The Laplace of probability density function of sojourn time into congestion is given by

$$\tilde{p}_c(s) = \pi_c V e^{z(x-X_L)} W e \quad (5.13)$$

where  $e$  is the unit column vector. The equilibrium probability vector  $\pi_u$  satisfies the following equations

$$\pi_u = \pi_c P_c \quad (5.14)$$

where  $P_c$  is given by

$$P_c = \tilde{p}(X_H, 0) \quad (5.15)$$

This gives the following relationship for  $\pi_c$  and  $\pi_u$ .

$$\pi_c = \pi_c P_C P_U \quad (5.16)$$

and

$$\pi_u = \pi_u P_U P_C \quad (5.17)$$

These combined with the normalization equations  $\pi_c.e=1$  and  $\pi_u.e=1$ , are used to obtain  $\pi_c$  and  $\pi_u$ . The above solution for probability density function has been obtained for the case  $NR_1 < C$ . A more generic result permitting  $C < NR_1 < NR$  may be obtained using Equations 4.52-4.55.

$$\tilde{p}(X_H, s) = \begin{bmatrix} V_1 e^{z_A(X_H - X_L)} V_1^{-1} & 0 \\ V_3 e^{z_A(X_H - X_L)} V_1^{-1} & 0 \end{bmatrix} \quad (5.18)$$

where  $z_A$  consist of negative eigenvalues of the key matrix  $D_1^{-1}(sI - M_1)$ . The eigenvalue matrix itself is given by Equation 5.19 where  $z_B$  constitutes of positive eigenvalues of  $D_1^{-1}(sI - M_1)$ . The solution for eigenvalues is given by (3.33) and the eigenvector  $V(s)$  is obtained using (3.34) and (3.35)

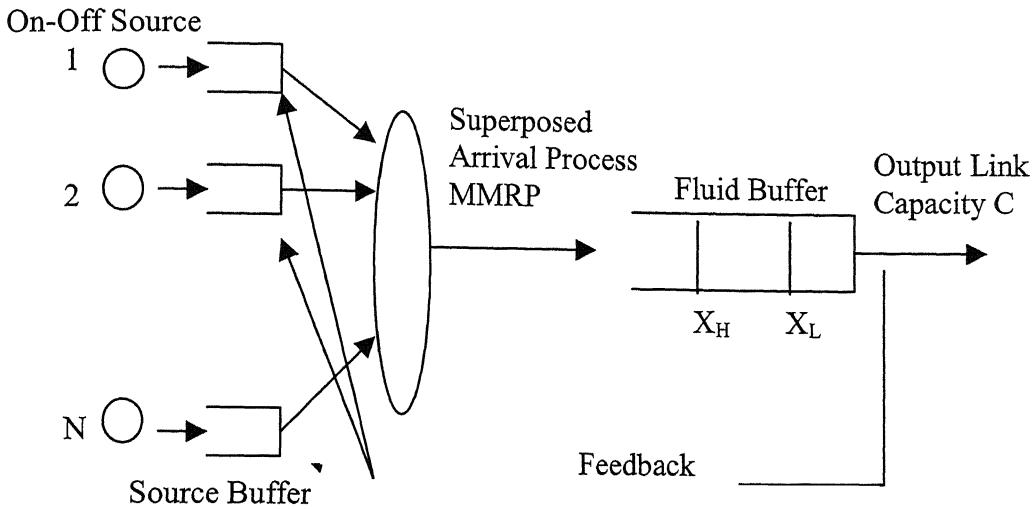
$$z(s) = \begin{bmatrix} z_A & 0 \\ 0 & z_B \end{bmatrix} \quad (5.19)$$

$$V(s) = \begin{bmatrix} V_1(s) & V_2(s) \\ V_3(s) & V_4(s) \end{bmatrix} \quad (5.20)$$

The magnitude of eigenvalues increase with decrease in On-Off source output rate (Equation 3.33). It is obvious from Equation 5.13 and 5.18 that increase in magnitude of negative eigenvalues result in decrease in mean sojourn time in the overload state. Thus reducing the regulated rate  $R_1$  which in turn results in reduction in mean sojourn time in congested state.

### 5.3 Feedback Controlled Sources with Source Buffers

In this section the statistical multiplexer is fed by sources with source buffer. Though this model does not have significant practical application but one such application is for implementing adaptive smoothening of burst for non-real time traffic. This allows the source to use source buffer to buffer the data till congestion is over. This is a kind of back-pressure mechanism. The real life back-pressure mechanism is far more complex than this simple model. It involves finite propagation delay and the multiplexer output has to be modeled as two-state Markov Process. A simplified model of back pressure mechanism involving finite propagation delay is discussed in Section 5.4.



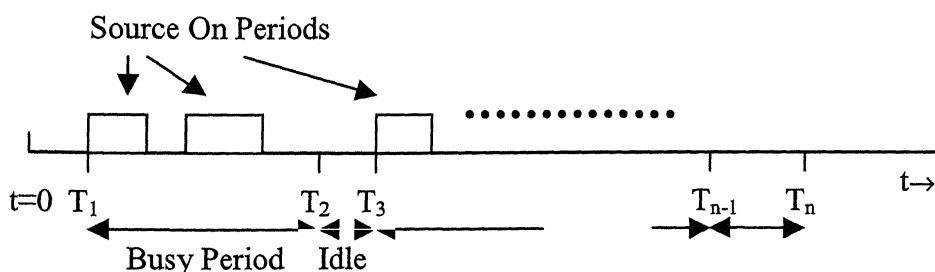
**Figure 5.3 Feedback Controlled On-Off Sources with Source Buffer (Model 2)**

The model under consideration is shown in Figure 5.3. A statistical multiplexer is fed by  $N$  On-Off sources. Each source generates fluid traffic at constant rate  $R$ . This traffic is fed to the source buffer. Source buffer empties at constant rate  $R$  when operating in uncontrolled mode. When the statistical multiplexer enters into congestion, it sends a binary feedback to the sources. On receiving the congestion signal from the multiplexer, the sources reduce the output rate of their source buffer to rate  $R_I$ . This results into a queue build up in the source buffer. It is assumed that rate  $R_I$  satisfies the stability condition (Equation 5.21). The source buffer is assumed to be infinite. Hence no loss takes place in buffering at source.

$$R_I > \lambda R / (\lambda + \mu) \quad (5.21)$$

When the multiplexer comes out of congestion state it sends a congestion clear signal to the sources. This results in source removing control over the output of the source buffer. While in the uncontrolled mode the fluid input to the multiplexer is simple superposition of fluid traffic from  $N$  On-Off Sources. In the controlled mode the output of the source buffer will show On-Off behavior but the On periods are no longer exponentially distributed. The off periods are exponentially distributed with mean period  $1/\lambda$  where  $1/\lambda$  is the mean off period of an On-Off source. For analytical simplicity the output of the source buffer in controlled mode is approximated as having exponentially distributed On and Off periods. When a source switches from the

controlled mode to uncontrolled output mode, if it is in On-state, then distribution of the first On period will be different from the other On-periods in the uncontrolled mode. This is because a source in On state has non-zero buffer contents in controlled state. This non-zero buffer contents in the source buffer result in extended first On period in uncontrolled state. For analytical simplicity the multiplexer model with feedback controlled sources with source buffer is approximated in the following manner. Multiplexer is fed by a fluid source with arrival process at the multiplexer given by MMRP. The modulating process in MMRP alternates between two transient Continuous Time Markov Chains given by infinitesimal generator  $M$  and  $M_l$ . It is assumed that the state of MMRP in overload period is given by number of source buffer outputs in On State. In underload period the state of MMRP gives the number of On-Off sources in On State. The transition between the two modulating Markov Chains take place when the multiplexer changes state from overload to underload state and vice-versa. In underload state the fluid arrival is governed by modulating process defined by CTMC  $M$  and in overload state by CTMC  $M_l$ . While there is a continuity in state when the MMRP switch from CTMC  $M$  to CTMC  $M_l$  at  $t = t_0$  (Figure 5.5), there is no such continuity when the MMRP transits from CTMC  $M_l$  to CTMC  $M$  at  $t = t_1$ . This is because the busy state of the output of the source buffer does not imply that the corresponding On-Off source is in On state. Hence if the MMRP is in state  $i$  just before the multiplexer enters the underload state, then the state  $j$  of the MMRP at  $t = t_1$  is obtained as follows.



**Figure 5.4 Busy Periods of Source Buffer Output**

A busy period of the source buffer output in controlled mode (region  $I$ ) constitutes of number of On and Off periods of the corresponding On-Off source (Figure 5.4). The transition from overload state to underload state is a random event. Given that the source buffer output was in On state, the probability the instant of

transition lies in On Period of the corresponding source is directly dependent on the ratio of source On and Off periods in a busy period of a source buffer. Hence given that a source buffer output is in On state, the probability  $\rho$  that the On-Off source is in On state is assumed to be given by

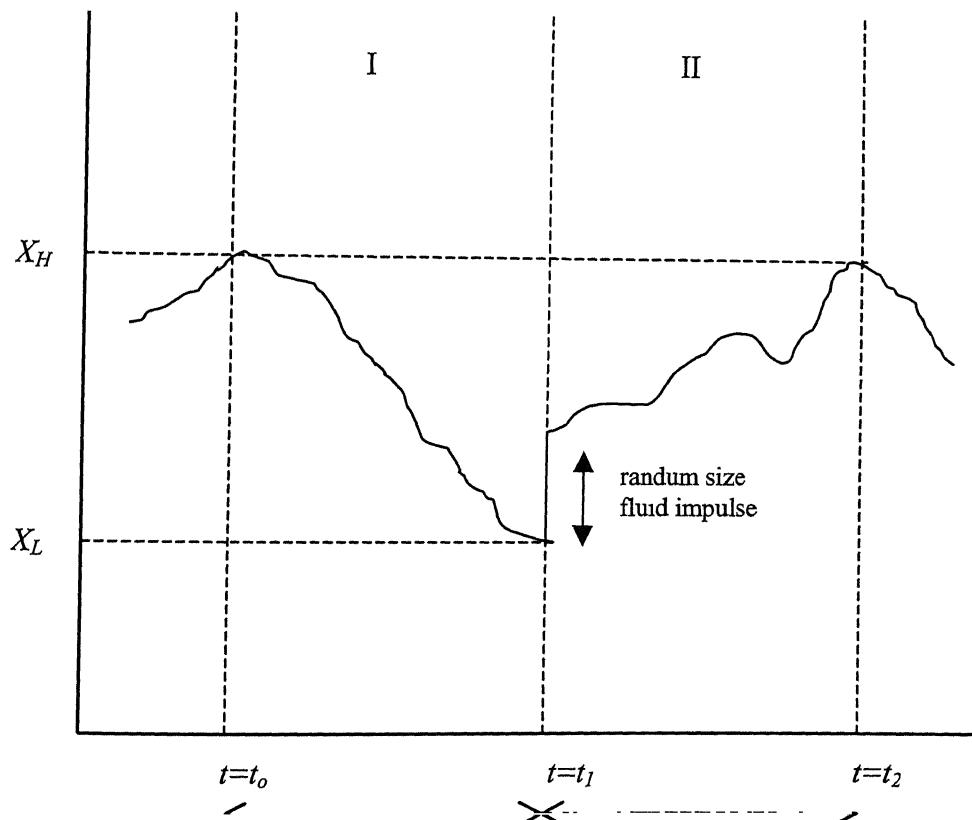
$$\rho = \frac{\lambda}{\lambda + \mu} \quad (5.22)$$

The transition matrix  $T$  giving transition from state  $i$  of  $M_I$  to state  $j$  of  $M$  is given by

$$T = \begin{bmatrix} t_{00} & \dots & t_{0N} \\ \vdots & \ddots & \vdots \\ t_{N0} & \dots & t_{NN} \end{bmatrix} \quad (5.23)$$

where the  $ij^{th}$  element of  $T$  is given by

$$t_{ij} = \begin{cases} \binom{i}{j} \rho^j (1 - \rho)^{i-j} & 0 \leq j \leq i \quad 0 \leq i \leq N \\ 0 & j > i \quad 0 \leq i \leq N \end{cases} \quad (5.24)$$



**Figure 5.5 A Sample Path of Fluid Buffer Contents with Time (Model 2)**

Next issue in the model is to incorporate the contents of source buffer at the instant of transition from overload state to underload state. In the approximate model it is assumed that all the source buffers release their fluid contents as fluid impulse at the instant of transition from controlled output mode to uncontrolled output mode. If the modulating process given by  $M_1$  is in state  $i$  at the instant just before the transition then the net fluid impulse to the multiplexer is sum of  $i$  homogeneous impulses from  $i$  active source buffer outputs. This approximation results in higher initial value buffer contents at the start of underload period. Hence the underload period obtained using this approximation should form an upper bound to the actual underload period. It is this approximate model that is analyzed in this section.

With these assumptions, the drift and infinitesimal generator for the fluid input to the statistical multiplexer are defined. Making use of the fluid conservation principle for the source buffer, the parameters for the process describing output flow from a source buffer are given in terms of parameters of an On-Off source. If On-Off source has mean On period  $1/\mu$  and mean Off period  $1/\lambda$  and peak Rate  $R$  then the output of the source buffer will also have mean On period  $1/\mu$  and mean Off period  $1/\lambda$  when operating in uncontrolled mode. In controlled mode with output rate of  $R_1$ , the conservation principle gives the mean On period  $1/\mu_1$  and mean Off period  $1/\lambda_1$  by

$$\lambda_1 = \lambda \quad (5.25)$$

$$\mu_1 = \frac{\mu R_1 - \lambda(R - R_1)}{R} \quad (5.26)$$

Since the rate  $R_1$  is such that the source buffer is stable hence  $1/\mu_1$  is always positive. The infinitesimal generator  $M$  for the MMRP in uncontrolled mode is given by

$$M \equiv \begin{bmatrix} -N\lambda & N\lambda & . & . & 0 \\ \mu & -(N-1)\lambda - \mu & (N-1)\lambda & . & 0 \\ . & . & . & . & . \\ 0 & . & . & . & \lambda \\ 0 & . & . & N\mu & -N\mu \end{bmatrix} \quad (5.27)$$

the drift  $D$  is given by

$$D \equiv \begin{bmatrix} -C & 0 & . & . & 0 \\ 0 & R-C & 0 & . & 0 \\ . & . & . & . & . \\ 0 & . & . & . & 0 \\ 0 & 0 & 0 & 0 & NR-C \end{bmatrix} \quad (5.28)$$

The infinitesimal generator  $M_1$  for controlled mode is given by

$$M_1 \equiv \begin{bmatrix} -N\lambda_1 & N\lambda_1 & . & . & 0 \\ \mu_1 & -(N-1)\lambda_1 - \mu_1 & (N-1)\lambda_1 & . & 0 \\ . & . & . & . & . \\ 0 & . & . & . & \lambda_1 \\ 0 & . & . & . & N\mu_1 - N\mu_1 \end{bmatrix} \quad (5.29)$$

and drift  $D_1$  by

$$D_1 \equiv \begin{bmatrix} -C & 0 & . & . & 0 \\ 0 & R_1 - C & 0 & . & 0 \\ . & . & . & . & . \\ 0 & . & . & . & 0 \\ 0 & 0 & 0 & 0 & NR_1 - C \end{bmatrix} \quad (5.30)$$

The density function for the impulse from the source buffer output is described next. It is assumed that the congestion duration is long enough to allow the source buffer to achieve steady state. The stationary probability density function for fluid contents in source buffer is given by

$$p(X = x) = \begin{cases} ae^{-bx} & x > 0 \\ 1 - \frac{a}{b} & x = 0 \end{cases} \quad (5.31)$$

where  $a$  and  $b$  are constants given by

$$a = \frac{\lambda R}{(\lambda + \mu)R_1} \cdot \frac{(\lambda + \mu)R_1 - \lambda R}{R_1(R - R_1)} \quad (5.32)$$

$$b = \frac{(\lambda + \mu)R_1 - \lambda R}{R_1(R - R_1)} \quad (5.33)$$

The net fluid impulse received at the statistical multiplexer at the instant of transition to underload state is sum of impulses with probability density given by Equation 5.31. The number of such impulses contributing to the fluid impulse at the multiplexer is dependent on state of MMRP just before the transition takes place. If  $Y_i$  represents the random variable giving the size of the impulse corresponding to state  $i$  of

the MMRP at the instant just before the transition from overload to underload state takes place, than the density function of  $Y_i$  is given by

$$p(Y_i = y) = \begin{cases} \sum_{j=1}^l \binom{i}{j} \left(1 - \frac{a}{b}\right)^{l-j} \frac{a^j y^{j-1} e^{-by}}{(j-1)!} & y > 0 \\ \left[1 - \frac{a}{b}\right]^l & y = 0 \end{cases} \quad (5.34)$$

The above definitions have been used to obtain the initial value of buffer contents at the beginning of underload period at time  $t = t_1$  in Figure 5.5. This will be sum of the low threshold value  $X_L$  and the random fluid impulse. The phase of the MMRP at the beginning of underload period is obtained in the following manner.

Let  $\pi_c$  and  $\pi_u$  be the row vectors giving equilibrium probability of the state of MMRP at the embedded points at the beginning of overload and underload periods respectively. Then

$$\pi_u = \pi_c P_c T \quad (5.35)$$

where  $P_c$  is given in terms of Laplace of probability density function for first passage to underload state  $\tilde{p}_c(x, s)$  by

$$P_c = \tilde{p}_c(X_H, 0) \quad (5.36)$$

Similarly if  $Y$  is the random variable denoting the size of the fluid impulse at the instant of transition from overload state to underload state then

$$\pi_c = \pi_u P_u \quad (5.37)$$

where  $P_u$  is given in terms of Laplace of probability density function for first passage to underload state,  $\tilde{p}_u(x, s)$ , and the probability density function,  $p_Y(y)$ , of the impulse size.

$$P_u = \int_0^{X_H - X_L} p_Y(y) \tilde{p}_u(X_L + y, 0) dy \quad (5.38)$$

Hence

$$\pi_c = \pi_c P_c T P_u \quad (5.39)$$

$$\pi_u = \pi_u P_u P_c T \quad (5.40)$$

These, combined with the normalization equations  $\pi_c.e=1$  and  $\pi_u.e=1$ , are used to obtain  $\pi_c$  and  $\pi_u$ . The Laplace of density function for first passage time to underload state,  $\tilde{p}_c(x,s)$ , is obtained using Equations 4.52-4.55. The key matrix for this is given by  $D_1^{-1}[sI - M_1]$  where  $D_1$  and  $M_1$  are defined earlier in this section. The first passage time to congestion,  $\tilde{p}_u(x,s)$ , is given by Equations 4.29, 4.30, 4.31, 4.32 with the key matrix given by  $D^{-1}[sI - M]$ .

At time  $t = t_1$ , there is finite possibility of the multiplexer going into congestion again if the fluid impulse is greater than  $X_H - X_L$ . The design of thresholds should be such that this probability is negligible. Each such sojourn into congested state is an independent random variable hence if mean sojourn time into overload state is given by  $\tau_o$ , then the mean congestion duration is given by  $\tau_o/(1-q)$  where  $q$  is the probability of the multiplexer entering into congestion at the instants corresponding to  $t = t_1$ . By keeping  $q$  negligible the mean congestion duration is given by the sojourn time into congested state.

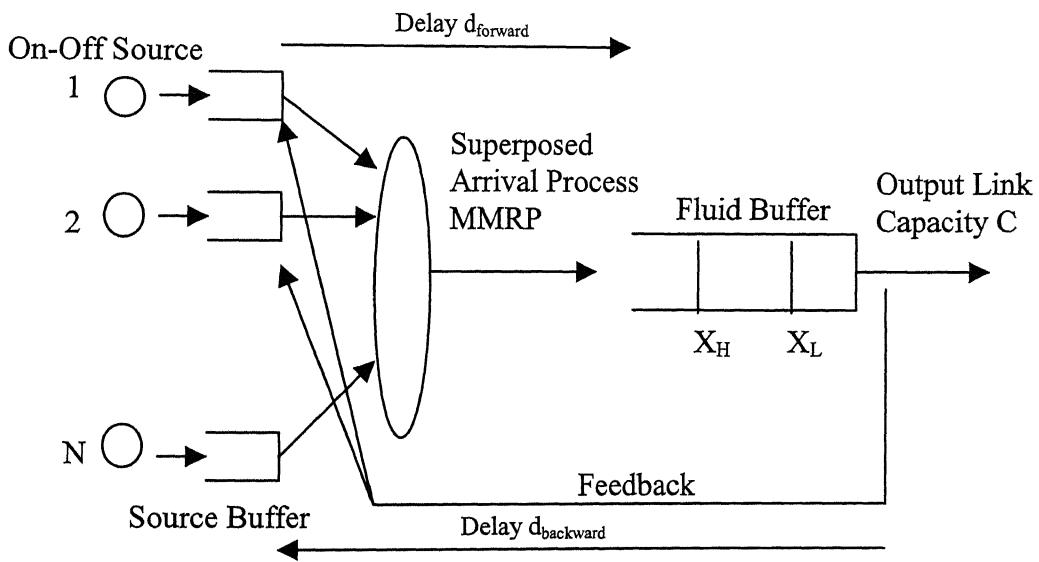
The mean congestion duration in multiplexer with feedback controlled sources with source buffer, will be higher than that for sources without source buffer. It will be higher than that for open-loop case also. The reason being that the eigenvalues decrease with increase in mean On-period (3.33). This results in higher mean for the sojourn time into congestion state. Thus applying feedback control with source buffer increases the congestion duration. The benefit of introducing feedback is that it reduces the overflow probability on the multiplexer buffer. These observations are verified in Section 5.5 through numerical examples.

## 5.4 Non-Negligible Feedback Propagation Delay

In this section the model of a statistical multiplexer with delayed feedback controlled sources is considered (Figure 5.6). This section develops the solution for obtaining the moments of overload and underload durations for this model. The model used in the previous section is extended to incorporate delayed feedback. Statistical multiplexer is once again modeled as a fluid buffer with constant rate output link. There are  $N$  On-Off type sources that feed input to the statistical multiplexer. All the  $N$  On-

Off sources use source buffer the output of which is regulated by feedback from statistical multiplexer. These source buffers allow the lossless flow of fluid when the statistical multiplexer is in congestion and the output flow from the source buffers is controlled to restrict the input to the statistical multiplexer.

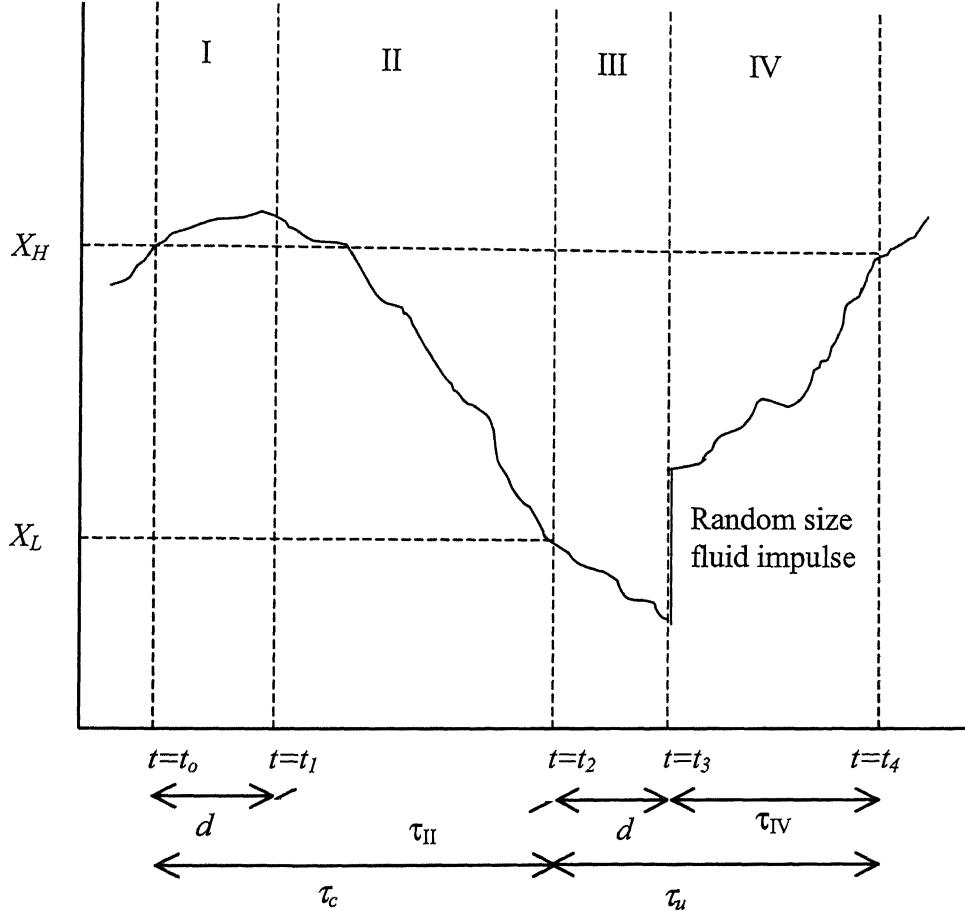
Let there be a guaranteed bandwidth associated with each source. When the multiplexer enters congestion it forces the sources to reduce the traffic to the guaranteed bandwidth thus allowing itself to recover from the congested state. For this, the multiplexer sends an explicit feedback signal to each of the sources. There is an associated non-negligible propagation delay in the backward path ( $d_b$ ) as well as forward path ( $d_f$ ). Thus the change in input flow due to feedback is observed at the input of the fluid multiplexer after a delay  $d_t$ , which is sum of propagation delay in forward and backward path (Equation 5.1). It is assumed that the processing delays at multiplexer or the On-Off sources are negligible.



**Figure 5.6 Feedback Model with Non-Negligible Propagation Delay**

The multiplexer sends the congestion free signal when it enters into underload state as described in Section 3.1. This feedback signal allows the sources to remove control on the output of the source buffer. Thus the sources start transmitting at their peak rate. This is reflected at the input of the multiplexer after a delay  $d_t$  of sending the congestion clear feedback signal to the input sources. The sources continue to operate in open loop mode till the multiplexer remains in underload state. When the contents in

fluid buffer of the multiplexer once again cross the high threshold mark, the multiplexer enters into congestion and sends the control signal to the sources. This once again forces the sources to control the output of their source buffers reducing it to the guaranteed bandwidth. This cycle continues.



**Figure 5.7 A Sample Path of Fluid Buffer Contents with Time (Model 3)**

The model defined above has wide applications. It can be modified to study the efficacy of backpressure mechanism to control the congestion in a node under consideration. It has application in design of adaptive traffic shapers and decision making in providing guaranteed bandwidth support to the sources. With no source buffer, the model discussed here may be used to study the performance of multimedia streaming with multirate codec support at the server.

The sojourn time analysis using the model described above has associated analytical complexity and may become intractable. To make the model analytically tractable, numbers of simplifying approximations have been made. Some of these

approximations have already been discussed in Section 5.3. First the On-Off sources have been assumed to be homogeneous and independent. The propagation delay has also been assumed to be same for all the sources. The assumption of homogeneous propagation delay allows us to model the input flow in the multiplexer as being modulated by two different chains, one in controlled mode and another in open loop mode. With heterogeneous propagation delay, making use of largest propagation delay would have resulted in upper bound for the sojourn time in congestion state. Hence the assumption of homogeneous propagation delay is not restrictive.

The round trip propagation delay is assumed to be exponentially distributed. This assumption is primarily for analytical ease. The results obtained with this assumption are compared with those obtained using simulation where the propagation delay is deterministic. The other assumption is that the multiplexer does not come out of congestion during the round trip delay interval. If the On and Off periods are comparable with round trip delay than this is a valid assumption. This assumption is also valid if the high and low thresholds are so chosen that the probability of the sojourn time in congestion state being less than the round-trip delay is negligible. Equation 4.89 shows dependence of density function of the sojourn time into congestion, on the difference between two threshold values.

The output process of the source buffer is once again approximated by On-Off process with exponentially distributed On and Off periods (Section 5.3). In uncontrolled output mode the mean On and Off periods are given by  $1/\mu$  and  $1/\lambda$  respectively. The output rate in uncontrolled mode is  $R$ . In controlled output mode, the output rate is reduced to  $R_1$ . Once again, for the sake of simplicity the On period is approximated as an exponentially distributed with mean parameters for On and Off periods ( $1/\mu_1$  and  $1/\lambda_1$  respectively) given by Equation 5.25 and 5.26

$$\lambda_1 = \lambda \quad (5.41)$$

$$\mu_1 = \frac{\mu R_1 - \lambda(R - R_1)}{R} \quad (5.42)$$

It is assumed that  $R_1$  satisfies the stability condition Equation 5.21 for the source buffer. With these approximations, the tools developed earlier in Section 4.1 and 4.2 are used to obtain the congestion duration and underload period for a multiplexer with feedback controlled sources and non-negligible propagation delay. The transition from

controlled state to uncontrolled state of source buffer results in release of fluid impulse in a similar manner as described in Section 5.3.

Figure 5.7 gives the definition of different intervals in a cycle of congestion and underload periods. At time  $t = t_0$  the multiplexer enters the state of congestion as the fluid buffer contents cross the high threshold of  $X_H$ . The fluid arrival process at the input of the multiplexer is described by the modulating Markov Chain with infinitesimal generator  $M$ . At time  $t = t_1$ , the feedback sent at time  $t = t_0$  results in the input arrival process being described by MMRP with infinitesimal generator  $M_I$ .

The sojourn time into congestion state is a sum of two variables, the exponentially distributed propagation delay with mean  $d_t$  and the first passage time to underload state given by the interval  $t = t_2 - t_1$ . Since the two are independent of each other, the moments of the two are independently obtained. The first passage time to underload is governed by the key matrix  $D_I^{-1}(sI - M_I)$  where  $D_I$  and  $M_I$  are as given below.

$$M_I \equiv \begin{bmatrix} -N\lambda_1 & N\lambda_1 & . & . & 0 \\ \mu_1 & -(N-1)\lambda_1 - \mu_1 & (N-1)\lambda_1 & . & 0 \\ . & . & . & . & . \\ 0 & . & . & . & \lambda_1 \\ 0 & . & . & N\mu_1 & -N\mu_1 \end{bmatrix} \quad (5.43)$$

$$D_I \equiv \begin{bmatrix} -C & 0 & . & . & 0 \\ 0 & R_I - C & 0 & . & 0 \\ . & . & . & . & . \\ 0 & . & . & . & 0 \\ 0 & 0 & 0 & 0 & NR_I - C \end{bmatrix} \quad (5.44)$$

The density function for the sojourn time in region II of Figure 5.7 is obtained using results of Section 4.1.2. One needs to find the density function for the buffer contents and the phase of the modulating process  $M$  at the instance  $t = t_1$ . Once again consider a process that assumes the value of the phase of the modulating process and contents of the fluid buffer at the instance where the arrival process changes from  $M$  to  $M_I$ . This process given by  $(M(t_n), X(t_n))$  forms an embedded Markov Chain. With its equilibrium probabilities giving the density function for the multiplexer buffer contents and the phase of the modulating process  $M$  at these embedded instances.

If  $f_X(x, t, x_0)$  is the transient probability density function for the fluid contents in the statistical multiplexer fed by fluid input defined by MMRP, given that at  $t = t_0$ , the buffer contents were at level  $x_0$ . Let  $\pi_c(x)$ ,  $\pi_I(x)$ ,  $\pi_u(x)$  and  $\pi_2(x)$  be the row vectors giving equilibrium probability of the state of MMRP at the embedded points  $t = t_0 +$ ,  $t = t_1 +$ ,  $t = t_2 +$  and  $t = t_3 +$  respectively (Figure 5.7). At  $t = t_0 +$  the fluid buffer content are at level  $X_H$ . At instant  $t = t_2 +$ , the fluid contents in the multiplexer buffer are at level  $x = X_L$ .

$$\pi_1(x) = \pi_c(X_H) P_I(x, X_H) \quad (5.45)$$

where  $P_I(x, X_H)$  is given by

$$P_I(x, X_H) = \int_0^{\infty} f_X(x, t, X_H) \cdot \frac{e^{-\frac{t}{d_t}}}{d_t} dt \quad (5.46)$$

$P_I(x, X_H)$  is given in terms of Laplace of the transient probability density function  $f_X(x, t, X_H)$

$$P_I(x, X_H) = \frac{1}{d_t} \tilde{f}_X(x, \frac{1}{d_t}, X_H) \quad (5.47)$$

The row vector  $\pi_u(X_L)$  is given by

$$\pi_u(X_L) = \int_{X_L}^{\infty} \pi_1(x) P_{II}(x) dx \quad (5.48)$$

where  $P_{II}(x)$  is given in terms of the Laplace of first passage time to underload state,

$\tilde{p}_c(x, s)$ , as

$$P_{II}(x) = \tilde{p}_c(x, 0) \quad (5.49)$$

Let  $\pi_{2-}(x)$  be the row vector giving state of MMRP and the contents of fluid buffer at instant  $t = t_2 -$ , then

$$\pi_{2-}(x) = \pi_u(X_L) P_{III}(x, X_L) \quad (5.50)$$

where  $P_{III}(x, X_L)$  is given by

$$P_{III}(x, X_L) = \int_0^{\infty} f_X^1(x, t, X_L) \cdot \frac{e^{-\frac{t}{d_t}}}{d_t} dt \quad (5.51)$$

$$P_{III}(x, X_L) = \frac{1}{d_t} \tilde{f}_X^1(x, \frac{1}{d_t}, X_L) \quad (5.52)$$

The transient probability density function is obtained in Appendix 5. The two transient probability density functions  $f_X(x, t, x_0)$  and  $\tilde{f}_X^1(x, t, x_0)$  differ in terms of the key matrix that governs their eigenvalues and eigenvectors. The key matrix for  $f_X(x, t, x_0)$  and  $\tilde{f}_X^1(x, t, x_0)$  are  $D^{-1}(sI - M)$  and  $D_1^{-1}(sI - M_1)$  respectively.

If  $Y_i$  represents the random variable giving the size of the impulse corresponding to state  $i$  of the MMRP at the instant just before the transition from overload to underload state takes place, than the density function of  $Y_i$  given by Equation 5.34 is used to obtain the row vector  $\pi_2(x)$ .

$$\pi_2(x) = \int_0^x \pi_{2-}(y) P_Y(x-y) T dy$$

(5.53)

where  $T$  is defined in Equation 5.23 and  $P_Y(y)$  is an  $(N+1)x(N+1)$  diagonal matrix whose  $i^{th}$  diagonal element is given by  $p(Y_i = y)$  as defined by Equation 5.34.

The row vector  $\pi_c(X_H)$  is given by

$$\pi_c(X_H) = \int_0^{X_H} \pi_2(x) P_{IV}(x) dx \quad (5.54)$$

where  $P_{IV}(x)$  is given in terms of the Laplace of first passage time to overload state,

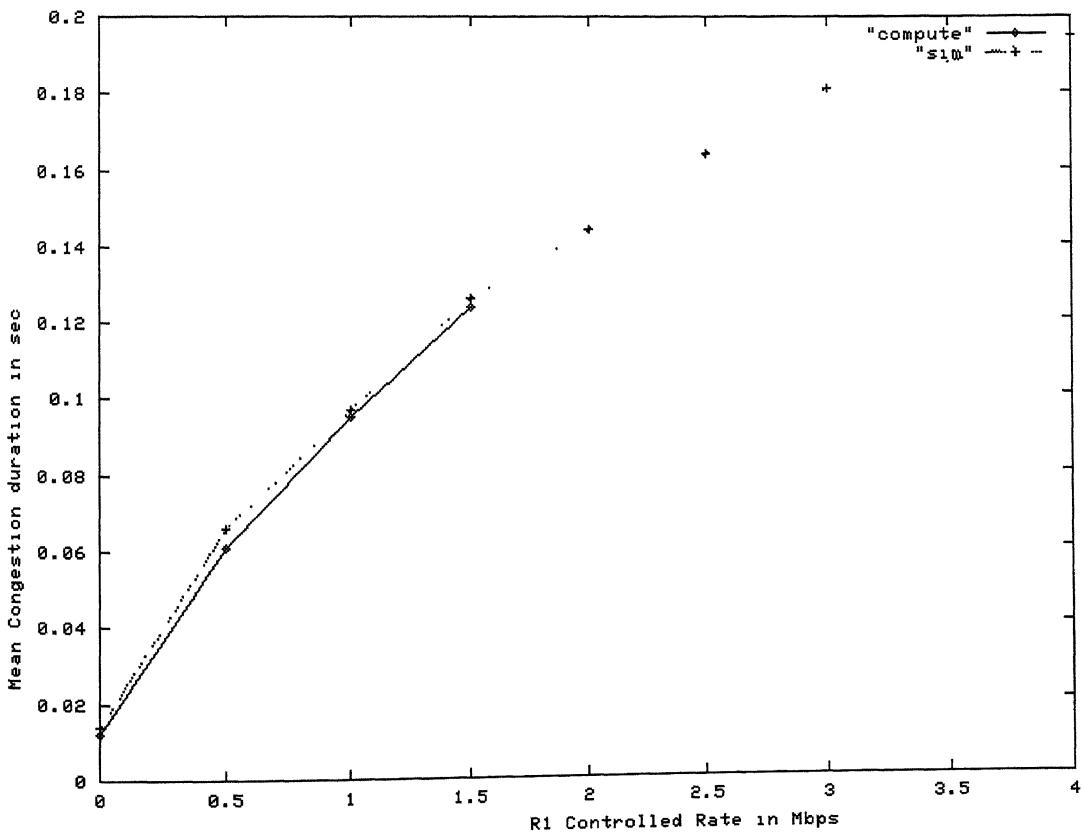
$\tilde{p}_u(x, s)$ , as

$$P_{IV}(x) = \tilde{p}_u(x, 0) \quad (5.55)$$

The mean congestion duration is then obtained as sum of mean propagation delay and mean duration of period  $II$  in Figure 5.7. Similarly the sojourn time into underload duration covering period  $III$  and  $IV$  in Figure 5.7 is given by sum of two independent random variables, the exponentially distributed round trip delay and the first passage time to congestion.

## 5.5 Numerical example

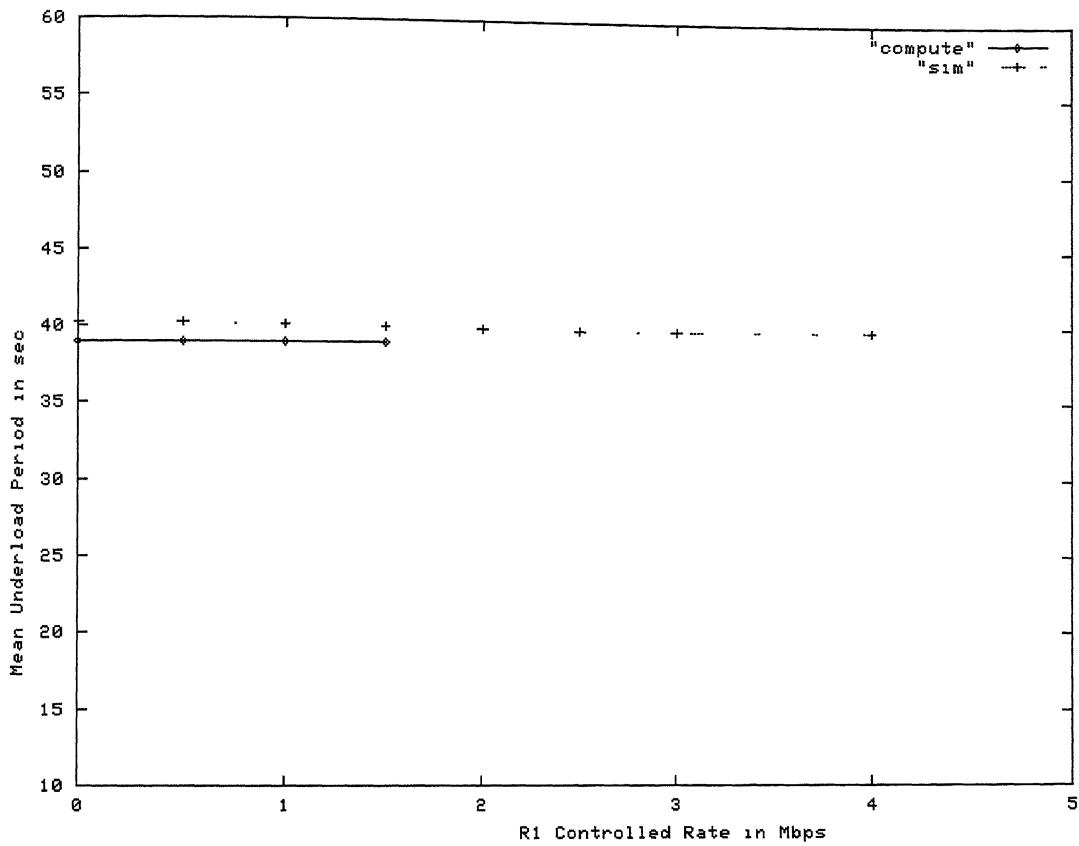
In this section we have taken an example to show the dependence of congestion duration and open-loop period on different traffic and system parameters. Numerical results are obtained using a sample example similar to the one used in Section 4.4. A fluid multiplexer is fed with 50 On-Off sources with mean On period of 100 ms and mean Off period of 200 ms. The multiplexer consists of a fluid buffer of size 3Mbits and has a link capacity of 85 Mbps. The high and low thresholds are fixed at 2 Mbits and 1 Mbit respectively. The results derived in this Chapter have been computed by approximating the superposed traffic at the input of the multiplexer, by a 2-state MMRP as described in Section 4.2. In open loop mode each source generates fluid at a rate of 4 Mbps when in On state. In controlled state the fluid rate is reduced to  $R_1$ . The controlled rate is varied keeping other parameters fixed.



**Figure 5.8 Mean Overload Period vs Controlled Rate  $R_1$  (Model 1)**

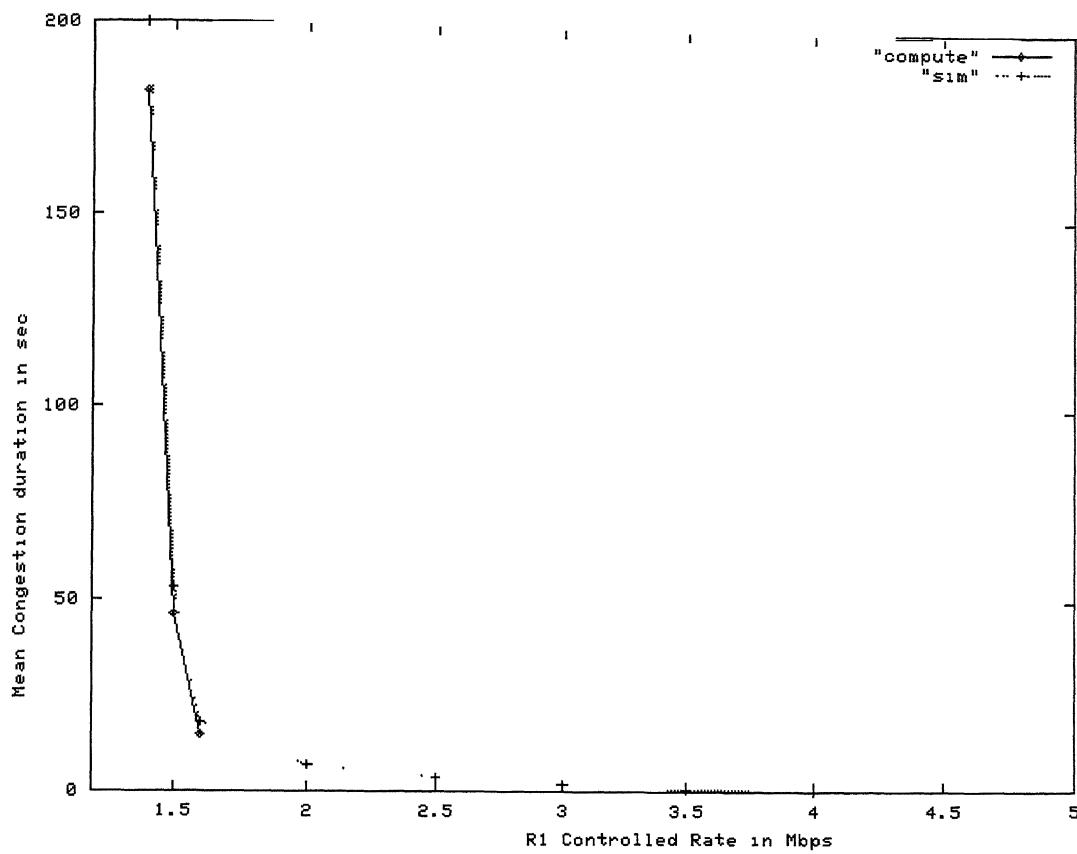
In Figure 5.8 the overload period is plotted against controlled rate. It is observed that the congestion duration reduces significantly with the reduction in controlled rate,

going to a minimum given by the difference in threshold levels and multiplexer link capacity. There is no impact on underload period (Figure 5.9).

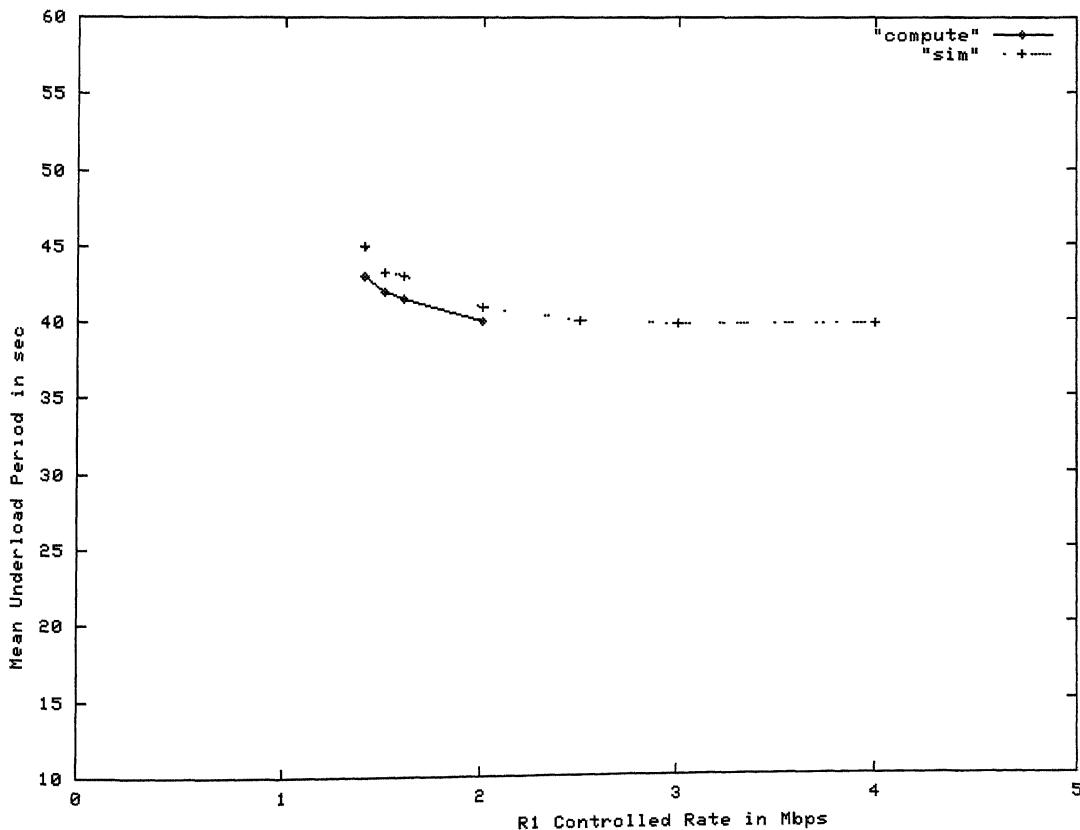


**Figure 5.9 Mean Underload Period vs  $R_1$  (Model 1)**

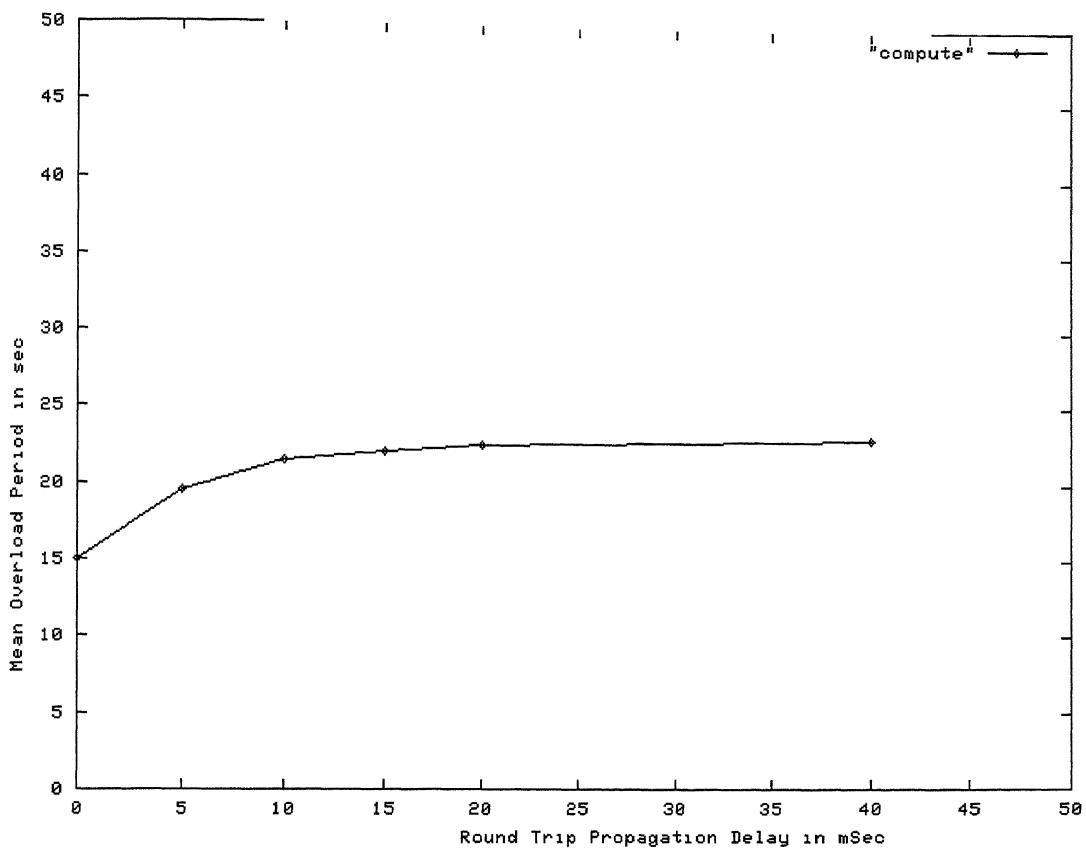
Figure 5.10 gives the variation in mean sojourn time in congestion as controlled rate is varied for Model 2. The source buffers are assumed to be of infinite size. For the simulation model, source buffer of 5 Mbits is used. This is almost of infinite size for a single source. It is observed that mean congestion duration increases exponentially as controlled rate approaches the mean rate at which On-Off source generates fluid. Underload period shows slight increase as the controlled rate is increased (Figure 5.11). This may be attributed to increased impulse at the beginning of an overload period as the mean buffer occupancy increases with decrease in output flow from source buffer. The difference between simulated and computed curve increases as the controlled rate is reduced. This is due to 2-state approximation. In case of single 2-state MMRP source a single source buffer is used. Hence the density function of fluid impulse is different from that due to superposition of  $N$  impulses.



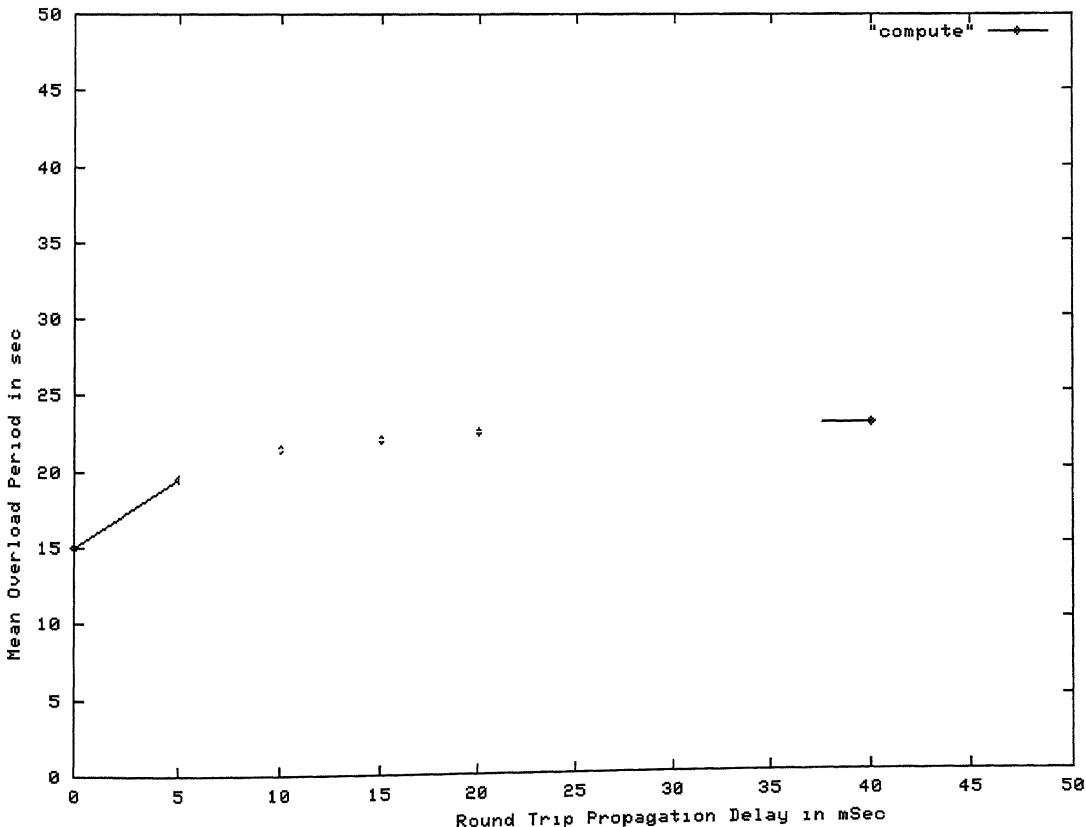
**Figure 5.10 Mean Overload Period vs  $R_1$  (Model 2)**



**Figure 5.11 Mean Underload Period vs  $R_1$  (Model 2)**

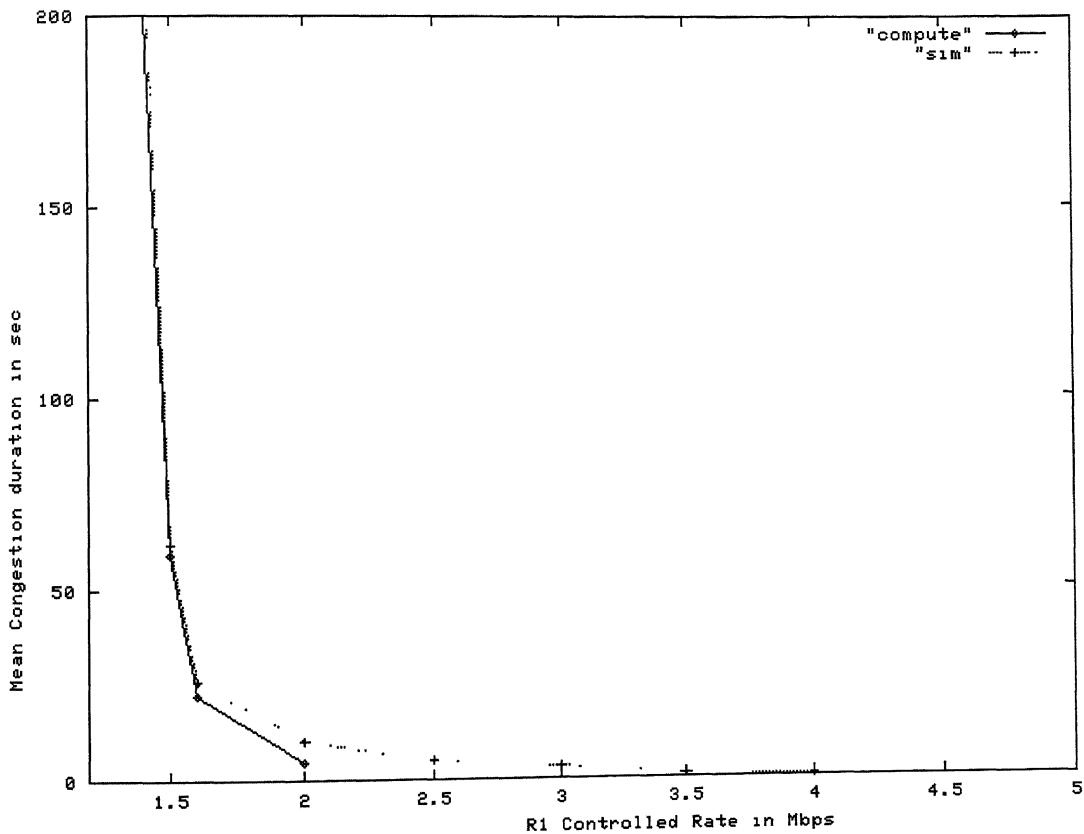


**Figure 5.12 Mean Overload Period vs Round Trip Propagation Delay (Model 3)**



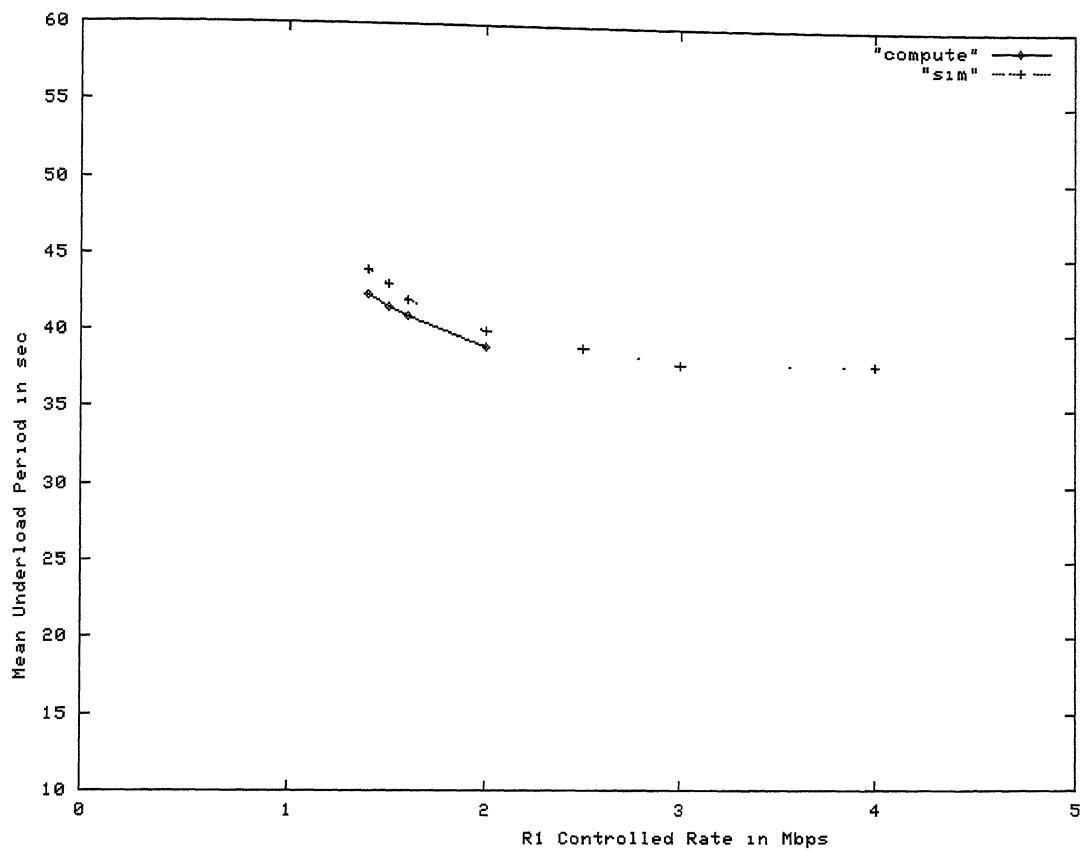
**Figure 5.13 Mean Underload Period vs Round Trip Delay (Model 3)**

The impact of non-negligible propagation delay in Model 3 is shown in Figure 5.12 and Figure 5.13. The example uses a regulated source buffer output rate of 1.6 Mbps in controlled mode. Rests of the parameters are same as those used in earlier examples. It is observed in Figure 5.12 that overload period increases significantly with increase in propagation delay. This is attributed to higher initial buffer content at which the control gets effective. On the other hand the underload period decreases slightly with increase in round trip propagation delay Figure 5.13. This is attributed to fluid level in multiplexer buffer falling below low threshold by the time the effect of removal of control on source is observed in the input flow to the multiplexer.

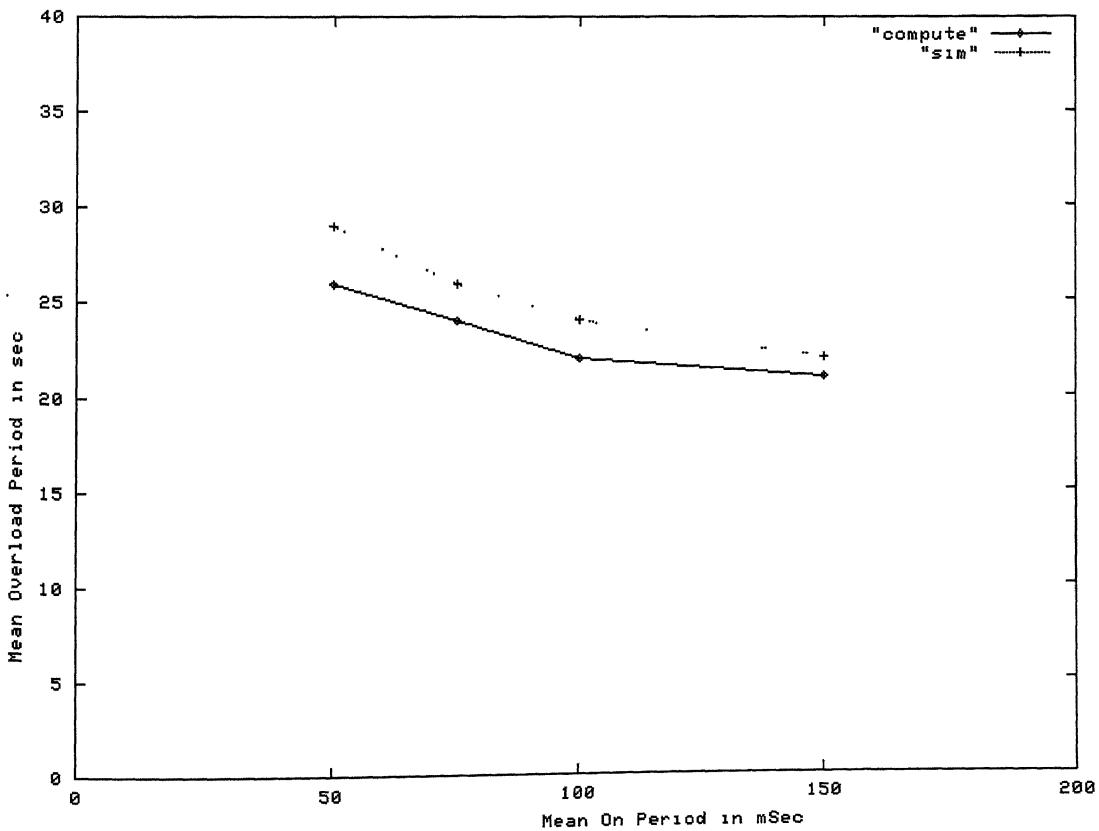


**Figure 5.14 Mean Congestion Duration vs  $R_1$  (Model 3)**

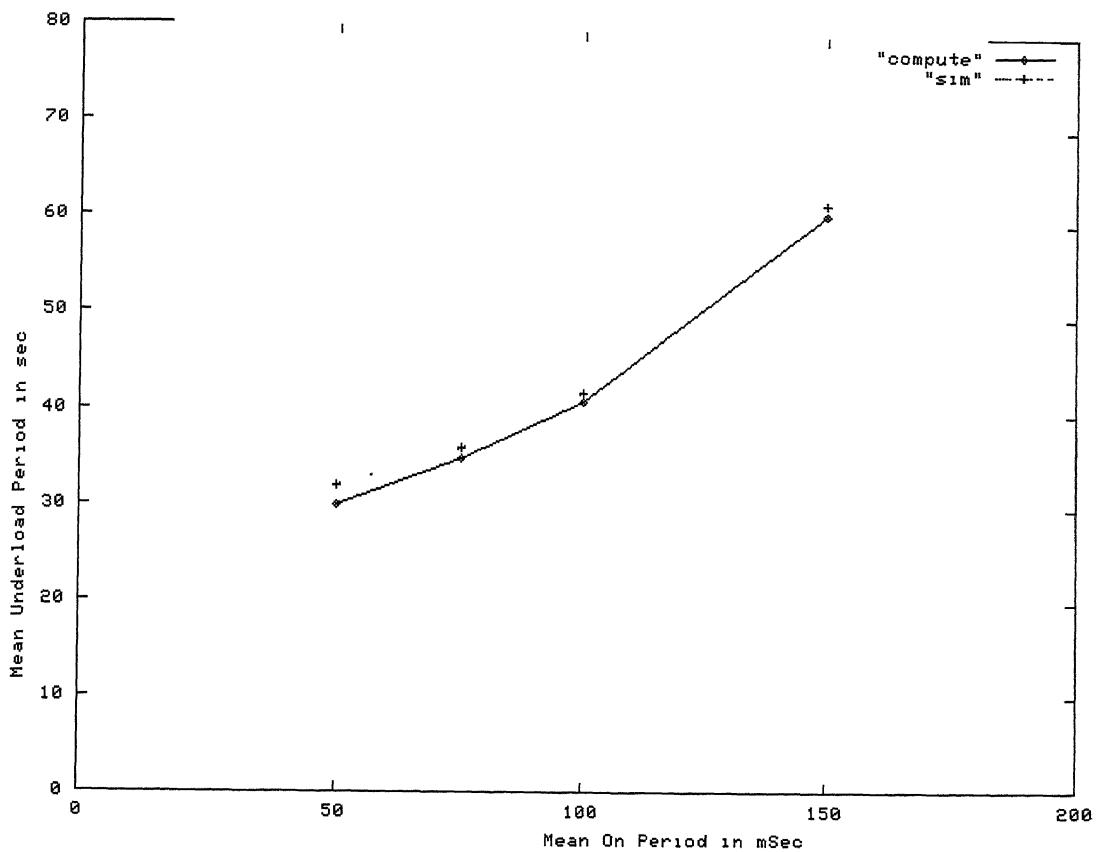
The variation of mean overload and underload periods with change in controlled output of source buffer for Model 3 is shown in Figure 5.14 and Figure 5.15 respectively. The round trip propagation delay is taken to be 20 ms. The results are similar to those in Figure 5.10 and Figure 5.11 for Model 2. Mean overload duration shows a drastic increase as  $R_1$  approaches the mean rate of On-Off source. Underload duration also shows a minor increase as  $R_1$  is reduced .



**Figure 5.15 Mean Underload Period vs  $R_1$  (Model 3)**



**Figure 5.16 Mean Overload Period Vs. Mean On Period of On-Off Source**



**Figure 5.17 Mean Underload Period vs Mean On Period of On-Off Source**

In Figure 5.16 and Figure 5.17 the variation in overload and underload periods is plotted against the change in burstiness of On-Off sources. The On and Off periods are varied so that the mean source utilisation remains constant. Thus without changing the mean fluid output of On-Off sources, the impact of change in On period is observed in Figure 5.16 and Figure 5.17. In this example the regulated source buffer output rate is plugged at 1.6 Mbps, and round trip propagation delay of 20 ms is used. While mean overload period decreases slightly with increase in On period, the mean underload period shows marked change. As the burstiness in On-Off source is reduced (increase on On period), the mean underload period also increases. This implies that the less bursty sources result in less frequent transitions to congestion.

## 5.6 Summary

In this chapter the sojourn time density function has been obtained for different models of fluid flow buffer with feedback-controlled sources. The model used in Section 5.4 has wide application and takes into account the non-negligible propagation delay. Analytical tractability of non-negligible propagation delay based sojourn time

analysis has been improved by making assumptions and approximations which are not restrictive in nature. The results demonstrate the effectiveness of regulation in reduction of congestion duration. It also demonstrates that use of source buffer increases mean congestion duration. Though the gain in form of lower loss probability is significant. The results obtained here provide a framework for Engineering of Call Admission Control for networks with hop-by-hop feedback control.

# Chapter 6

## Call Admission Control

Call Admission Control is an important function in Traffic Management. It prevents congestion within the network by controlling the number of active connections, thus sustaining acceptable Quality of Service (QoS) for all connections. The call admission function uses the traffic description provided by the user at call setup time, to accept or reject the call based on whether the resources are available to guarantee the QoS to the existing users as well as to the new connection request.

In this chapter we have developed a procedure for use in Call Admission Control based on effective bandwidth. The effective bandwidth approach to Call Admission Control is based on the fact that a notional bandwidth can be assigned to each source which reflects its burstiness and the queuing parameters of the server. It has been seen that for efficient network utilization and stringent performance requirements under highly bursty traffic, large size buffers are required. Hence the effective capacity based on asymptotic decay rates of steady state distributions of buffer occupancy has been suggested as one of the metric for call admission control.

In the effective bandwidth methods proposed in the literature [ELW93] [GUE91] [KES93] [CHO96], the tail probability  $P[X > X_B]$  is used for obtaining the effective bandwidth of the source. In most of the cases the loss probability has been approximated by

$$P_{Loss} \sim e^{-bX_B} \quad (6.1)$$

where  $b$  is a constant and  $X_B$  is the buffer size. This results in additive effective bandwidth i.e. the effective bandwidth of the combined traffic to an ATM server is the sum of the effective bandwidths of the individual sources. These effective bandwidths give good result when the source utilization (ratio of mean cell rate to peak cell rate) is high, but the results are conservative for low source utilization. This is because the multiplexing effect of the bursty sources is not accounted for.

In this chapter a methodology for obtaining more realistic value of effective bandwidth is proposed [MUD95]. This method also takes into account the number of existing connections for computing the effective bandwidth. Thus the multiplexing

effect of the bursty sources is taken into consideration when admitting a new call into the network. This chapter starts with the description of the model and analytical approach used in this thesis, giving justification for the approach chosen. The stationary probability distribution for the contents of the fluid buffer has been obtained using the methodology based on embedded content process at the epochs when the arrival process changes states. This approach is different from the approach proposed in [ANI82] based on spectral decomposition of the key matrix  $D^{-1}M$ . This approach is presented next, followed by an algorithm to obtain the asymptotic loss probability in the following form:

$$P(Q > B) \sim Ae^{-bX_B} \quad (6.2)$$

Here  $A$  is the asymptotic constant dependent upon factors like the number of connections passing through the multiplexer, and other queuing parameters like link capacity. Finally another approach has been provided for obtaining the asymptotic constant  $A$  in Equation 6.2. The results thus obtained have been compared with those provided in [GUE91] and with exact computations based on [ANI82].

## 6.1 The Model

The Model used in this Chapter is same as that described in Section 3.1. The statistical multiplexer is represented by a fluid buffer with a constant rate output channel. The fluid buffer is assumed to be of infinite size. The fluid arrival rate  $r(t)$  is modulated by a background stochastic process. This modulating process governs the rate at which the fluid flows into the fluid buffer. As discussed in Section 3.1 the modulating process is assumed to be a finite Markov chain, unless specified otherwise. The instantaneous fluid generation rate from such a source is dependent on the state of the modulating Markov chain. The arrival process is called a Markov Modulated Rate Process.

Let  $N + 1$  be the number of states in the background Markov chain  $M(t)$  defined over state space  $[0, N]$ . When the modulating process is in state  $i \in [0, N]$ , the fluid is generated at a rate  $r_i$ . Let  $X(t)$  represent the quantity of fluid in fluid buffer at time  $t$ . The fluid flows out of the fluid buffer at a constant rate  $c$ . Hence the output rate from the buffer  $r_o(t)$  is given by

$$r_o(t) = \begin{cases} c & X(t) > 0 \text{ or } r_i(t) \geq c \\ r_i(t) & \text{otherwise} \end{cases} \quad (6.3)$$

The Markov Modulated Rate Process defining the fluid arrival at the fluid buffer is once again assumed to a superposition of  $N$  homogeneous, independent On-Off fluid sources. An On-Off source has already been discussed in Section 3.1.

### 6.1.1 On-Off Source Description

|                         |             |       |
|-------------------------|-------------|-------|
| Peak Rate               | $R$         |       |
| Mean Source Utilization | $\rho$      |       |
| Mean Rate               | $\rho R$    | (6.4) |
| Mean On period          | $1/\mu$     |       |
| Mean Off period         | $1/\lambda$ |       |

The source utilization factor  $\rho$  is defined as

$$\rho = \frac{\lambda}{\lambda + \mu} \quad (6.5)$$

## 6.2 Stationary Probability Distribution

In this section an approach for obtaining the stationary probability distribution for the content of an infinitely large fluid buffer fed by  $N$  On-Off sources is presented. The model has been described in Section 6.1 and Section 3.1. The On and Off periods have been assumed to be exponentially distributed. The buffer is emptied by a constant capacity output link. The superposed arrival process is a fluid arrival process with the modulating ( $N+1$ ) state birth and death process given by  $M(t)$ . If  $X(t)$  represents the content of the fluid buffer at time  $t$ , then we define a stochastic process  $(M(t), X(t))$ . In this approach the stationary distribution of the buffer content process at epochs when the modulating process  $M(t)$  changes state has been obtained. The continuous-time stationary distribution is obtained from the distribution for the discrete time Markov Process. Appropriate approximation allow the simple method for obtaining the asymptotic tail probability and effective bandwidth [MUD95].

The dynamics of the fluid buffer is given as follows:

1. When the input arrival rate  $r_i$  is greater than the output channel capacity  $C$ , the buffer content increases at a constant drift  $d_i=r_i-C$
2. When the input fluid rate  $r_i$  is less than the output capacity  $C$ , the buffer content decreases at a constant drift  $d_i=r_i-C$ , if the buffer is not empty. If the buffer is empty it remains empty.

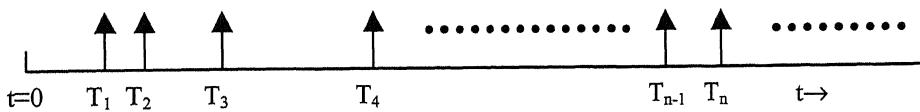
Thus the stationary probability of this system  $(M(t), X(t))$  exists provided the mean drift is negative and the system is stable.

Let  $\pi_i$ ,  $i \in [0, N]$ , be the stationary state probabilities of the modulating birth and death process  $M(t)$ . Then the stability condition requires

$$\sum_{i \in [0, N]} \pi_i r_i < C \quad (6.6)$$

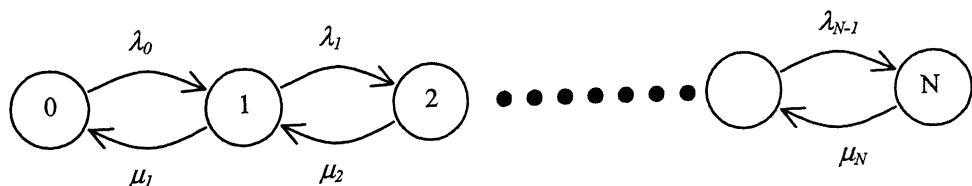
Let  $\{T_1, T_2, \dots, T_n, \dots\}$  be the random instants when the background process  $M(t)$  changes its state Figure 6.1. The arrival process is piece-wise constant and right continuous with almost surely increasing jump times  $\{T_n : n \geq 1\}$ .

$$0 < T_1 < T_2 < \dots < T_n < T_{n+1} \dots$$



**Figure 6.1 The Epochs of Jumps in the Background Process**

Let  $(M(T_n), X(T_n))$  defines the state of the system at the instant  $t=T_n$ . In this discussion  $t=T_n$  is the instant just before the jump in the background process takes place. Then  $(M(T_n), X(T_n))$  forms a discrete-time Markov Process. Let  $(P(X(T_n) \leq x | M(T_n)=i))$  be the conditional distribution for the content of the fluid buffer at the state change epoch  $T_n$ . At this instant the background process jumps from state  $i$  to next state  $k$  in accordance with the one-step transition for the modulating Markov chain. Assuming the background process to be the Birth and Death Process given by the Figure 6.2, if the modulating process is in state  $i$  in the time interval  $[T_{n-1}, T_n)$ , then the modulating process  $M(t)$  would be in state  $(i+1)$  or  $(i-1) \in [0, N]$  in the time interval  $[T_n, T_{n+1})$ .



**Figure 6.2 Background Birth and Death Process**

Let  $Z_n$  be a random variable denoting the net fluid that potentially gets added or removed in the sojourn period  $[T_{n-1}, T_n]$  when the modulating process is in state  $i$ . The sojourn periods are assumed to be independent of each other but are dependent on state  $i$  of the modulating chain in the the  $n$ th sojourn period,  $[T_{n-1}, T_n]$ . For the deterministic fluid input in the time interval  $[T_{n-1}, T_n)$  the variable  $Z_n$  can be given as

$$Z_n = \frac{T_n - T_{n-1}}{r_i - C} \quad n > 1 \quad (6.7)$$

Hence  $Z_n$  may vary over the positive real axis or negative real axis depending on whether drift  $d_i$  is positive or negative.  $Z_n$ ,  $n \in (1, \infty)$  are independent variables and are only dependent on state  $i$  of the modulating chain. For the background process already stated, the sojourn periods are exponentially distributed with the state dependent mean and hence the probability density function  $g_i(z)$  for  $Z(i)$  may be given as

For  $i$  s.t.  $r_i > C$

$$P(Z(i) = z) = \begin{cases} \left(\frac{\lambda_i + \mu_i}{r_i - C}\right) e^{-\left(\frac{\lambda_i + \mu_i}{r_i - C}\right)z} & z \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (6.8)$$

For  $i$  s.t.  $r_i < C$

$$P(Z(i) = z) = \begin{cases} \left(\frac{\lambda_i + \mu_i}{C - r_i}\right) e^{-\left(\frac{\lambda_i + \mu_i}{C - r_i}\right)z} & z \leq 0 \\ 0 & \text{otherwise} \end{cases} \quad (6.9)$$

Here it is assumed that  $r_i \neq C$ . At time  $T_n$ , the modulating Markov Process  $M(t)$  jumps to state  $i$  through either  $(i-1) \rightarrow i$  transition or  $(i+1) \rightarrow i$  transition (Figure 6.2). The probability of either of the transitions, given that a jump to state  $i$  has taken place, may be found from the transition frequencies of the two jumps. These are dependent on transition rates of the two jumps. Given that a transition to state  $i$  has taken place, then  $\alpha'_{i+1}$  is the probability that the previous state was  $(i+1) \in [0, N]$  and  $\alpha'_{i-1}$  denotes the probability that the transition given was  $(i-1) \rightarrow i \in [0, N]$ . Then these two probabilities are given by

$$\alpha'_{i-1} = \frac{\pi_{i-1} \lambda_{i-1}}{\pi_{i-1} \lambda_{i-1} + \pi_{i+1} \mu_{i+1}} \quad (6.10)$$

$$\alpha'_{i+1} = \frac{\pi_{i+1} \mu_{i+1}}{\pi_{i-1} \lambda_{i-1} + \pi_{i+1} \mu_{i+1}} \quad (6.11)$$

where  $\pi_i$   $i \in [0, N]$  is the stationary probability of the Markov Chain of Figure 6.2. The probability distribution for the discrete-time Markov Process  $(M(T_n), X(T_n))$  may now be obtained as follows:

For  $i \in [0, N]$  s.t.  $r_i > C$

$$P_{n+1}(X \leq x / M = i) = \begin{cases} \alpha'_{i-1} \int_0^x P_n(X \leq (x-z) / M = i-1) \cdot g_i(z) dz + \\ \alpha'_{i+1} \int_0^x P_n(X \leq (x-z) / M = i+1) \cdot g_i(z) dz \end{cases} \quad (6.12)$$

For  $i \in [0, N]$  s.t.  $r_i < C$

$$P_{n+1}(X \leq x / M = i) = \begin{cases} \alpha'_{i-1} \int_{-\infty}^0 P_n(X \leq (x-z) / M = i-1) \cdot g_i(z) dz + \\ \alpha'_{i+1} \int_{-\infty}^0 P_n(X \leq (x-z) / M = i+1) \cdot g_i(z) dz \end{cases} \quad (6.13)$$

The stationary probability distribution  $P_i(x)$  for the discrete-time Markov Process  $(M(T_n), X(T_n))$  may be obtained from the limiting probabilities

$$P_i(x) \equiv P(X \leq x / M = i) \equiv \lim_{n \rightarrow \infty} P_n(X \leq x / M = i) \quad (6.14)$$

Thus the stochastic integrals for the stationary distribution of the embedded process may be written as:

For  $i \in [0, N]$  s.t.  $r_i > C$

$$P_i(x) = \alpha'_{i-1} \int_0^x P_{i-1}(x-z) \cdot g_i(z) dz + \alpha'_{i+1} \int_0^x P_{i+1}(x-z) \cdot g_i(z) dz \quad (6.15)$$

For  $i \in [0, N]$  s.t.  $r_i < C$

$$P_i(x) = \alpha'_{i-1} \int_{-\infty}^0 P_{i-1}(x-z) \cdot g_i(z) dz + \alpha'_{i+1} \int_{-\infty}^0 P_{i+1}(x-z) \cdot g_i(z) dz \quad (6.16)$$

The above given set of stochastic integrals are convolution equations and may be represented in Laplace Transform form. Assuming that the transforms exist for  $g_i(z)$  defined for  $z$  in the negative half of the real axis, we get the following set of equations:

Let

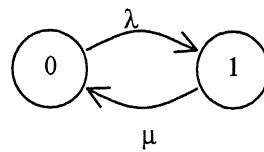
$$\tilde{P}_i(s) \equiv \int_{-\infty}^{\infty} P_i(x) e^{-sx} dx \quad (6.17)$$

$$\tilde{G}_i(s) \equiv \int_{-\infty}^{\infty} g_i(z) e^{-sz} dz \quad (6.18)$$

Then Equation 6.15 and Equation 6.16 may be written as

$$\tilde{P}_i(s) = \alpha'_{i-1} \tilde{P}_{i-1}(s) \tilde{G}_i(s) + \alpha'_{i+1} \tilde{P}_{i+1}(s) \tilde{G}_i(s) \quad i \in [0, N] \quad (6.19)$$

This is the standard Wiener-Hopf equation in Laplace transform form and exact solution for the same may be obtained by using the Wiener-Hopf factorization method [SPE85] [NOB58]. In this thesis we continue with the integral form of equations for the discrete-time Markov Chain to develop the procedure for obtaining the asymptotic loss probability of a fluid buffer fed by multiple homogeneous On-Off Sources. First we obtain the exact solution for an infinite fluid buffer fed by a single On-Off Source, starting from the set of equations defined above (Equation 6.15, Equation 6.16).



**Figure 6.3 The 2-state Discrete-time Markov Chain**

### 6.2.1 Fluid Buffer with Single On-Off Source

Consider a fluid buffer fed by a single On-Off Source with exponentially distributed On and Off periods. The rest of the parameters are same as those defined in Section 3.1. In the On period of the source the buffer gets filled at a deterministic rate R-C. It is emptied at the constant rate C when source is in Off state. The stationary probability of the two-state discrete-time Markov Chain (Figure 6.3) is given by

$$\pi_0 = \frac{\mu}{\lambda + \mu} \quad (6.20)$$

$$\pi_1 = \frac{\lambda}{\lambda + \mu} \quad (6.21)$$



**Figure 6.4 Points on which the Embedded Process is Defined**

The stationary probability distribution of the embedded Markov Process  $P(X \leq x)$  may be obtained from:

$$P(X \leq x) = \pi_0 P_0(x) + \pi_1 P_1(x) \quad x \geq 0 \quad (6.22)$$

where  $P_i(x)$   $i \in [0,1]$  is as defined earlier,  $P(X \leq x / M=i)$ .

The probability densities  $g_0(z)$  and  $g_1(z)$  are given by

$$g_1(z) = \begin{cases} \left(\frac{\mu}{R-C}\right) e^{-\left(\frac{\mu}{R-C}\right)z} & z \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (6.23)$$

$$g_0(z) = \begin{cases} \left(\frac{\lambda}{C}\right) e^{\left(\frac{\lambda}{C}\right)z} & z \leq 0 \\ 0 & \text{otherwise} \end{cases} \quad (6.24)$$

For the fluid buffer fed by single On-Off source, Equation 6.15 and Equation 6.16 may now be written as

$$P_1(x) = \int_0^x P_0(x-z) g_1(z) dz \quad (6.25)$$

$$P_0(x) = \int_{-\infty}^0 P_1(x-z) g_0(z) dz \quad (6.26)$$

Since the form of solution for  $P_i(x)$  is a sum of exponential terms, we may write them as

$$P_0(x) \equiv 1 - a_{00} e^{-b_0 x} - a_{01} e^{-b_1 x} \quad x \geq 0 \quad (6.27)$$

$$P_1(x) \equiv 1 - a_{10} e^{-b_0 x} - a_{11} e^{-b_1 x} \quad x \geq 0 \quad (6.28)$$

Using Equation 6.27 and Equation 6.23 in Equation 6.25 and then comparing it with Equation 6.28 we obtain the following relationships:

$$a_{10} = -\frac{\mu \cdot a_{00}}{(R-C)(b_0 - \mu/(R-C))} \quad (6.29)$$

$$a_{11} = -\frac{\mu \cdot a_{01}}{(R-C)(b_1 - \mu/(R-C))} \quad (6.30)$$

$$a_{10} + a_{11} = 1 \quad (6.31)$$

The boundary value condition  $P_1(0)=0$  also results in relationship given by Equation 6.31. Similarly applying Equation 6.28 and Equation 6.24 in Equation 6.26 and comparing with Equation 6.27 we get

$$a_{00} = \frac{\lambda \cdot a_{10}}{C(b_0 + \lambda/C)} \quad (6.32)$$

$$a_{01} = \frac{\lambda \cdot a_{11}}{C(b_1 + \lambda/C)} \quad (6.33)$$

From Equation 6.29, Equation 6.32 and Equation 6.30, Equation 6.33 it can be seen that  $b_i, i \in [0, N]$  satisfies the following relationship

$$\frac{\mu}{(R-C)} \cdot \frac{\lambda}{C} \cdot \frac{1}{(b_i - \mu/(R-C))(b_i + \lambda/C)} = -1 \quad (6.34)$$

which gives the two values of  $b_i$ .

$$b_i = 0, \left( \frac{\mu}{(R-C)} - \frac{\lambda}{C} \right) \quad i \in [0, 1] \quad (6.35)$$

Applying the following boundary condition to obtain the coefficients  $a_{10}, a_{11}$ .

$$\lim_{x \rightarrow \infty} P_i(x) = 1 \quad i \in [0, 1] \quad (6.36)$$

If  $b_0=0$  then we get

$$a_{00} = a_{10} = 0$$

$$a_{11} = 1$$

$$a_{01} = \frac{\lambda}{\mu} \cdot \frac{(R-C)}{C}$$

and

$$P(x) = 1 - \frac{\lambda R}{(\lambda + \mu)C} e^{-x((\lambda + \mu)C - \lambda R)/C(R-C)} \quad x \geq 0 \quad (6.37)$$

which is same as that obtained in [ANI82] and given in Appendix 1. This exercise was done to show the equivalence of the results obtained by different approaches. Next we obtain the similar relationships for the case of fluid buffer with the arrival process being superposition of  $N$  homogeneous sources. This relationship is used in Section 6.2.3 to obtain the approximation for the asymptotic constant in Equation 6.2.

## 6.2.2 Fluid Buffer with N On-Off Sources

Once again the generic model given in Section 6.1 is used for a fluid buffer being fed by the superposition of  $N$  On-Off sources. For the system where the combined arrival process is superposition of  $N$  homogeneous On-Off sources with peak rate  $R$ , the drift parameters and the parameters of infinitesimal generator  $M$  are given by:

For  $i \in [0, N]$

$$\begin{aligned} r_i &= iR \\ \lambda_i &= (N - i)\lambda \\ \mu_i &= i\mu \end{aligned} \tag{6.38}$$

The stationary probability for the discrete-time Markov Chain (Figure 6.2) have the following relationship obtained using the balance equation.

$$\pi_{i+1} = \frac{(N - i)\lambda}{(i + 1)\mu} \pi_i \quad i \in [0, N] \tag{6.39}$$

$$\sum_{i=0}^N \pi_i = 1 \tag{6.40}$$

Once again expressing the solution of stationary probabilities  $P_i(x)$  as the sum of  $(N+1)$  exponentials and continuing with Equation 6.15 and Equation 6.16, we obtain the relationship governing the coefficients in the general solution of  $P_i(x)$ .

For  $i \in [0, N]$

$$P_i(x) = P(X \leq x / M = i) \equiv 1 - \sum_{j=0}^N a_{ij} e^{-b_j x} \quad x \geq 0 \tag{6.41}$$

The probability density function  $g_i(z)$  for the potential net fluid inflow or outflow in a sojourn period when the background process is in state  $i$ , is obtained from Equation 6.8 and Equation 6.9 as

For  $i > \lfloor C/R \rfloor$

$$g_i(z) = \begin{cases} \left(\frac{(N-i)\lambda + i\mu}{iR - C}\right) e^{-\left(\frac{(N-i)\lambda + i\mu}{iR - C}\right)z} & z \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{6.42}$$

For  $i \leq \lfloor C/R \rfloor$

$$g_i(z) = \begin{cases} \left(\frac{(N-i)\lambda + i\mu}{C - iR}\right) e^{-\left(\frac{(N-i)\lambda + i\mu}{C - iR}\right)z} & z \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{6.43}$$

Using Equation 6.41 in Equation 6.15 and Equation 6.16 along with the definitions of  $g_i(z)$  and comparing the terms on RHS with those on LHS we obtain the following relationships:

For  $i > \lfloor C/R \rfloor$

$$a_{ij} = -\frac{(N-i)\lambda + i\mu}{(b_j(iR-C) - ((N-i)\lambda + i\mu))} (\alpha_{i+1}^l a_{(i+1)j} + \alpha_{i-1}^l a_{(i-1)j}) \quad (6.44)$$

$$\sum_{j=0}^N a_{ij} = 1 \quad (6.45)$$

For  $i \leq \lfloor C/R \rfloor$

$$a_{ij} = \frac{(N-i)\lambda + i\mu}{(b_j(iR-C) - ((N-i)\lambda + i\mu))} (\alpha_{i+1}^l a_{(i+1)j} + \alpha_{i-1}^l a_{(i-1)j}) \quad (6.46)$$

This set of relationship may be used to obtain the  $a_{ij}$  in terms of  $a_{Nj}$  for all  $i < N$ . The characteristic polynomial that is satisfied by all  $b_j$  may also be obtained using the above given system of equations. The roots of the  $(N+1)th$  order polynomial will give all the coefficient  $b_j$ ,  $j \in [0, N]$ .

For  $i=N$ , Equation 6.44 gives

$$a_{Nj} = -\frac{N\mu}{[b_j(NR-C) - N\mu]} \alpha_{N-1}^N a_{(N-1)j} \quad (6.47)$$

$$\Rightarrow a_{(N-1)j} = -\frac{[b_j(NR-C) - N\mu]}{N\mu} \cdot \frac{a_{(N-1)j}}{\alpha_{N-1}^N} \quad (6.48)$$

For  $N > i > \lfloor C/R \rfloor$ , Equation 6.44 leads to

$$a_{(i-1)j} = -\frac{[b_j(iR-C) - ((N-i)\lambda + i\mu)]}{(N-i)\lambda + i\mu} \cdot \frac{a_{ij}}{\alpha_{i-1}^i} - \frac{\alpha_{i+1}^i a_{(i+1)j}}{\alpha_{i-1}^i} \quad (6.49)$$

For  $0 < i < \lfloor C/R \rfloor$ , Equation 6.47 gives

$$a_{(i-1)j} = \frac{[b_j(iR-C) - ((N-i)\lambda + i\mu)]}{(N-i)\lambda + i\mu} \cdot \frac{a_{ij}}{\alpha_{i-1}^i} - \frac{\alpha_{i+1}^i a_{(i+1)j}}{\alpha_{i-1}^i} \quad (6.50)$$

For  $i=0$ , (6.46) results in

$$a_{0j} = \frac{N\lambda}{(-b_jC - N\lambda)} a_{1j} \quad (6.51)$$

Let  $a_{ij} = k_{ij} a_{Nj}$  where  $k_{ij}$  is a constant defined for  $i, j \in [0, N]$ . Define another constant  $l_{ij}$  such that

For  $i > \lfloor C/R \rfloor$

$$l_{ij} = -\frac{[b_j(iR-C) - ((N-i)\lambda + i\mu)]}{(N-i)\lambda + i\mu} \quad (6.52)$$

For  $i < \lfloor C/R \rfloor$ ,

$$l_y = \frac{[b_j(iR - C) - ((N-i)\lambda + i\mu)]}{(N-i)\lambda + i\mu} \quad (6.53)$$

Hence Equation 6.48, 6.49 and 6.50 result in the following iterative formula

$$k_{(N-1)j} = l_{Nj} \quad (6.54)$$

$$k_{(i-1)j} = k_{ij} \frac{l_{ij}}{\alpha'_{i-1}} - k_{(i+1)j} \frac{\alpha'_{i+1}}{\alpha'_{i-1}} \quad 0 < i < N \quad (6.55)$$

$$k_{0j} = \frac{k_{1j}}{l_{0j}} \quad (6.56)$$

Equating the value of  $k_{0j}$  from Equation 6.55 and Equation 6.56, the characteristic polynomial in  $b$  may be obtained, the roots of which will give the coefficients  $b_j$ . As shown in [ANI82], the characteristic polynomial for this kind of system will have  $N_1$  roots with positive real part corresponding to the number of states with positive drift ( $(iR-C) > 0$ ). Similarly there will be  $N_2$  roots, (such that  $N_1 + N_2 = N+I$ ), with negative real part corresponding to the number of states with negative drift ( $(iR-C) < 0$ ). Applying the boundary condition as  $x \rightarrow \infty$ , one gets  $a_y = 0$  for all roots  $b_j$  which have negative real part. Using  $a_y = k_{ij}a_{Nj}$  in Equation 6.45 one gets  $N_1$  simultaneous equations in  $N_1$  variables  $a_{Nj}$  for all  $j$  for which  $b_j$  has positive real part. This defines the complete system of equations from which the stationary probability can be obtained.

The stationary probability distribution for the content in the fluid buffer is given by

$$P(X \leq x) = \sum_{i=0}^N \pi_i P_i(x) \quad (6.57)$$

In this chapter we have used the approach provided above to obtain the approximate asymptotic loss probability. This approach is given in Section 6.2.3. The iterative procedure given here is more efficient than the one given in [ANI82] where the asymptotic constant requires computation of  $N_1$  stable eigenvalues of the key matrix.

### 6.2.3 Approximation for the loss probability

Here we approximate the solution of stationary probabilities  $P_i(x)$  by a single exponential term corresponding to the dominant root  $b_N$  in Equation 6.41.

For  $i \in [0, N]$

$$P_i(x) = P(X \leq x / M = i) = 1 - a_{iN} e^{-b_N x} + o(x) \quad x \geq 0$$

We assume here that  $a_{Nj}$  for  $j \neq N$  are small compared with  $a_{NN}$ . Hence we approximate  $a_{NN}$  to be unity. Next we use (6.54), (6.55) and (6.56) to obtain the loss probability after obtaining the approximate  $P_i(x)$  from these equations. This loss probability will form a close upper bound for the asymptotic loss probability and may be used for real time Call Admission Control procedure.

$$P_{loss} = \sum_{i=0}^N \pi_i a_{iN} e^{-b_N x} \quad (6.58)$$

The asymptotic decay rate  $b_N$  or dominant root of the characteristic polynomial is explicitly given in [KOS74], [ANI82] as

$$b_N = \frac{N((\mu + \lambda)C - N\lambda R)}{C(NR - C)} \quad (6.59)$$

## 6.3 Alternate Approach for Asymptotic Loss

### Probability

The exponential term in the expression for asymptotic buffer occupancy distribution depends on the source characteristics. While the buffer provides the smoothening effect that is captured by the decay rate constant, the asymptotic constant captures the multiplexing effect as has been mentioned earlier. With the simplifying assumption of  $A$  being unity, the effective bandwidths of all the sources add up. If  $C$  is the equivalent bandwidth of a single On-Off source then the effective bandwidth of  $N$  multiplexed sources will be  $N.C$ . For a bufferless model, Equation 6.2 gives

$$P_{loss} \sim A \quad (6.60)$$

Thus the asymptotic constant is the probability that the instantaneous aggregate inflow rate  $r$  is greater than the output channel capacity  $C$ . Hence the loss probability in the case of fluid buffer may be given by

$$P_{loss} \sim P(r > C).e^{-bx} \quad (6.61)$$

Here  $b$  is the dominant asymptotic decay rate given by Equation 6.59. The distribution of the flow rate  $r$  may be obtained from the stationary distribution of the underlying Markov Chain formed by the superposition of sources. In the case of superposition of  $N$  homogeneous, two state On-Off sources

$$P(r > C) = \sum_{i=\lfloor C/R \rfloor}^N \binom{N}{i} \rho^i (1-\rho)^{N-i} \quad (6.62)$$

A still easier and simple approach for modeling the asymptotic constant is by approximating the discrete distribution of  $r$ . Where the instantaneous input rate  $r$  takes only discrete values, it can be approximated by a suitable envelop provided the number of sources is large. This is the case of ATM networks. This approximation of a discrete distribution of  $r$  by an envelop allows simple approximate formula which may be better suited for CAC. In [MUD95] the aggregate fluid rate was approximated by the Gaussian envelop. It is a realistic assumption if large number of sources with similar parameters are multiplexed together. Each source is characterized by its average bit rate and standard deviation.

## 6.4 Call Admission Control

A general Call Admission Control function is based on obtaining the loss probability for the finite fluid buffer. A call is accepted if the loss probability due to the existing as well as the new connection is less than the loss probability requirements of all the connections. The criteria based on effective bandwidth method requires allocation of notional bandwidth  $C_i$  to a connection. The Call Admission Function uses the following rule for accepting a new connection

$$\sum_{i=0}^N C_i \leq C_{Link} \quad (6.63)$$

where  $C_{Link}$  is the output link capacity.

In the effective bandwidth methods proposed in the literature, this notional bandwidth is independent of the number of connections. We have used the loss probability given in Section 6.2.3 and 6.3 for obtaining the notional bandwidth and it changes as the number of connection increases.

Algorithm:

Step 1: Obtain the Asymptotic constant using a close estimate for the effective bandwidth  $\hat{C}_N$  for  $N$  sources including the new source. Use this estimate for obtaining asymptotic constant using either of the approaches given in Section 6.2.3 and 6.3.

Step 2: Obtain the effective bandwidth of  $N$  sources using the following formula:

$$C = \frac{NR\alpha - N(\mu + \lambda) + \sqrt{(N(\mu + \lambda) - NR\alpha)^2 + 4N^2\lambda R\alpha}}{2\alpha} \quad (6.64)$$

where  $\alpha$  is given by

$$\alpha = \frac{\ln(A/\varepsilon)}{x} \quad (6.65)$$

Here  $\varepsilon$  is the loss probability and  $x$  is the buffer size.

The choice of approximation for the effective bandwidth in step 1 decides whether the effective bandwidth obtained in Step 2 is on conservative side or not. If  $C_E$  is the exact effective bandwidth of  $N$  sources. Then  $\hat{C}_N > C_E$  will result in higher value of asymptotic constant  $A$  and hence the conservative value of  $C_N$  in step 2. The choice of  $\hat{C}_N < C_E$  will result in the value of  $C_N$  in step 2 which is lower than  $C_E$ . Hence if the effective capacity  $C_{N-1}$  for  $N-1$  connections is known, then the estimated effective bandwidth for  $N$  connections has been approximated by

$$\hat{C}_N = N.C_{N-1}/(N-1) \quad (6.66)$$

This is used to obtain the effective bandwidth of  $N$  sources from Step 2 above. This effective bandwidth is used to accept or reject the new call.

#### Heterogeneous sources:

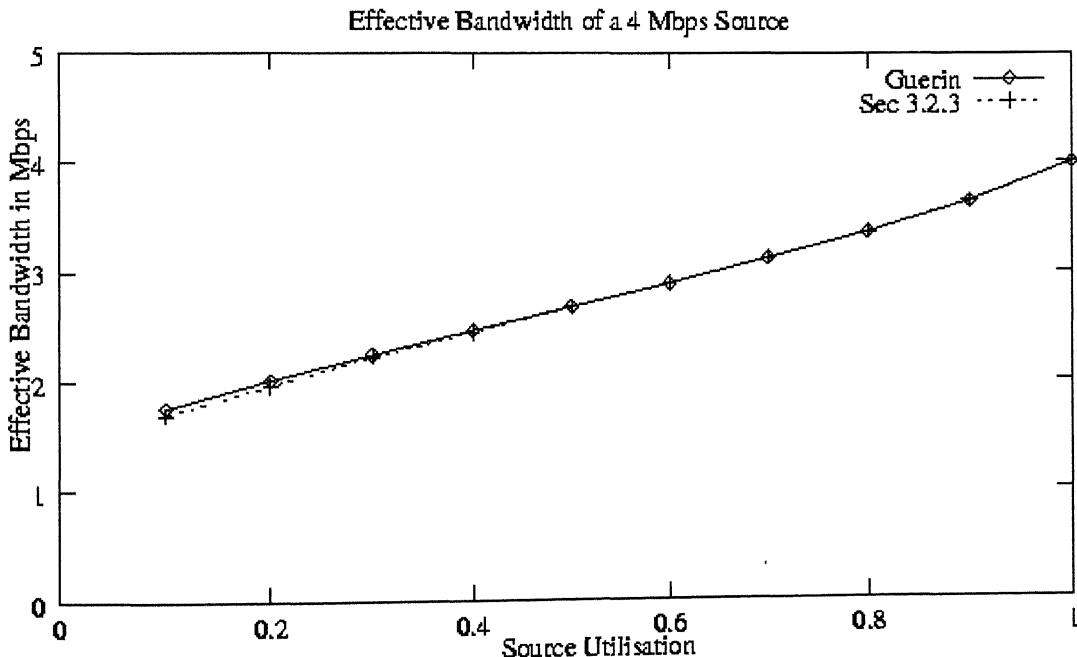
For heterogeneous sources forming multiple traffic classes the effective bandwidth has been obtained using segregation approach. In this approach the effective bandwidth of each class of traffic is obtained and the combined effective bandwidth is given by sum of the effective bandwidth of each traffic class. Segregation approach once again overestimates the link capacity as it does not take into account the multiplexing effect across different traffic classes. The approach given in Section 6.3 permits the computation of combined effective bandwidth of all the connections using the algorithm proposed above. The asymptotic constant  $A$  is obtained using the estimated effective bandwidth and the effective bandwidth is computed using the

following equation for Step 2. This has been obtained using the explicit form for dominant root given in [KOS74].

$$C = \sum_{i=1}^N \frac{R_i \alpha - (\mu_i + \lambda_i) + \sqrt{((\mu_i + \lambda_i) - R_i \alpha)^2 + 4\lambda_i R_i \alpha}}{2\alpha} \quad (6.67)$$

## 6.5 Numerical Results

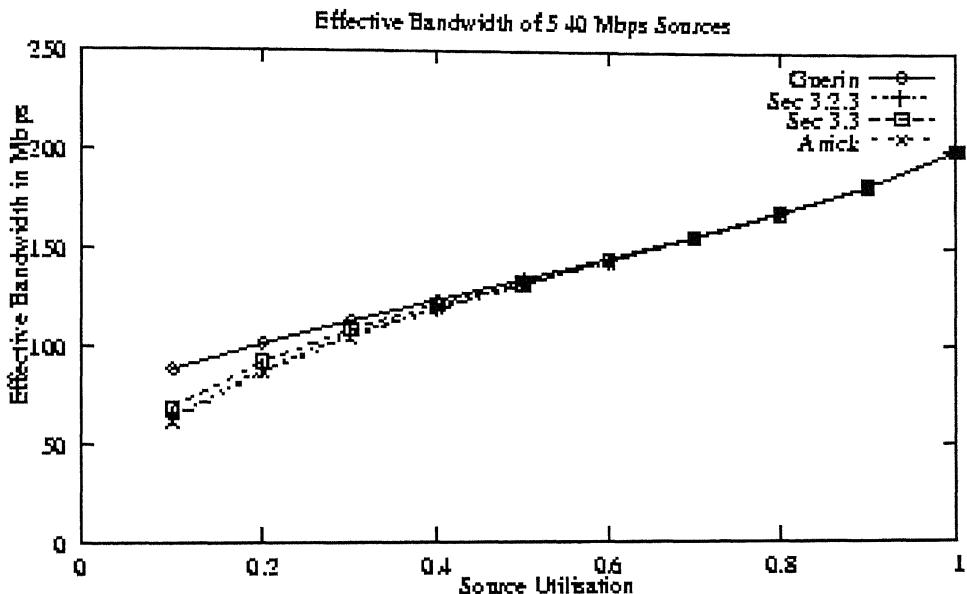
In this section, we provide some numerical examples to illustrate the effective bandwidth concept. A comparison of the results obtained from the algorithm developed in this Chapter are compared with those obtained in [ANI82] and [GUE91]. The results have been obtained with the need to demonstrate the gains achieved in terms of resource utilisation by adopting the algorithm given in this Chapter. The discussion in this section will also highlight the limitation if any of the effective bandwidth method proposed here. The numerical examples have been chosen so as to demonstrate the gains and accuracy of the approach for wide range of situations.



**Figure 6.5 Equivalent Capacity Vs Source Utilisation (Single Source)**

In Figure 6.5 the effective bandwidth of a single source has been plotted for different values of source utilization  $\rho$ . Source utilisation has been taken as a parameter for burstiness of the source traffic. The On-Off source has a mean burst length of 100ms. In the On period the source generates traffic at the rate of 4 Mb/s which is fed

to the fluid buffer of size 3 Mbits. The mean off period of the source varies with the source utilisation keeping the mean On-Period constant. The output link with capacity equivalent to effective bandwidth of the source guarantees the loss probability of less than  $10^{-5}$ . The results plotted in Figure 6.5 clearly show that for a single source the effective bandwidth obtained from the approach presented in this chapter exactly overlaps the effective bandwidth obtained using the exact calculation given in [ANI82] and the fluid flow approximation of [GUE91].

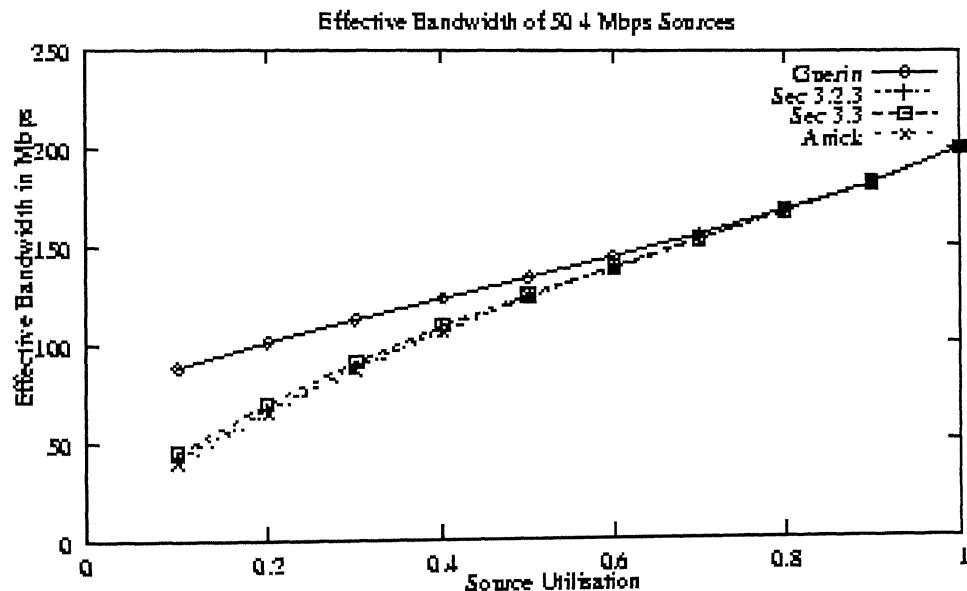


**Figure 6.6 Equivalent Capacity Vs. Source Utilisation (5 Sources)**

The multiplexing behavior is captured by the plots of Figure 6.6. Five On-Off sources feed a buffer of size 3 Mbits. The Peak rate of each source is 40 Mb/s and the mean burst length is 10 ms. Once again the effective bandwidth for the combined traffic is obtained for the loss probability of  $10^{-5}$ . The effective bandwidth of the combined traffic is plotted against the source utilisation  $\rho$ . For the higher source utilisation corresponding to low burstiness, the effective bandwidth curve obtained using algorithm presented in this chapter overlaps the one obtained through computation based on exact asymptote. But as the burstiness of the source increases (low source utilisation), the plots clearly show that the effective bandwidth obtained by the approach presented in this chapter are closer to exact computation in comparison with that obtained using fluid flow approximation in [GUE91]. For low source utilisation the gain in resource utilisation is of the order of 50%. This gain will be more

as the number of sources would increase. It is clear that the one based on [GUE91] or Equation 6.1 results in very poor utilisation of resources.

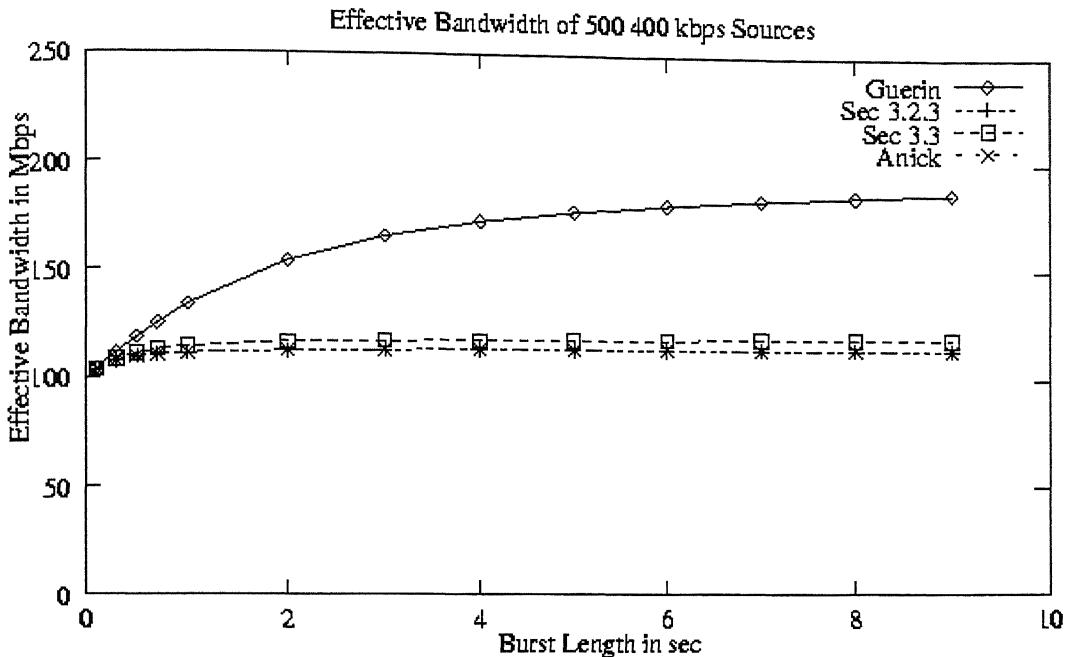
The gain in resource utilisation due to multiplexing gain is more evident in Figure 6.7. Here the number of sources used is 50. In this example 50 On-Off sources of peak rate 4 Mb/s and mean burst period of 100ms feed the fluid buffer of size 3 Mbits. Once again the effective bandwidth obtained using different approaches is plotted against source utilisation. As is evident from Figure 6.7, the effective bandwidth obtained using algorithm in this paper forms a close upper bound resulting in a much better resource utilisation than the one based on fluid flow approximation of [GUE91]. Once again it is clear from the figure that the multiplexing gain is lost when the burstiness decreases. As the source utilisation increases, the curves almost converge and overlap in the region of low burstiness.



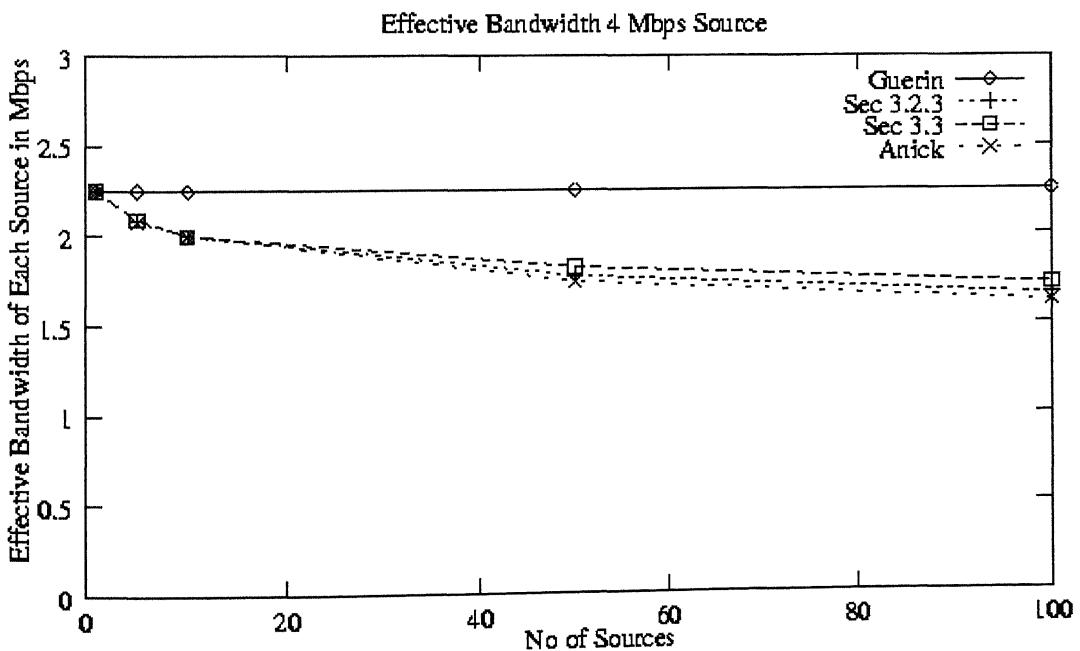
**Figure 6.7 Equivalent Capacity Vs. Source utilisation (50 Sources)**

The dependence of equivalent capacity on burst length is shown in Figure 6.8. Here 500 On-Off sources fed a fluid buffer of capacity 3Mbits. The peak rate of each source is 400 kbps and the loss probability of  $10^{-5}$  is assumed. The effective bandwidth increases with the increase in burst length. Once again the results obtained using the approach presented here, are much closer to the exact computation than with those obtained in [GUE91]. As the burst length increases keeping the source utilisation constant, the multiplexing gain increases as is evident from Figure 6.8. The results

based on Section 6.3 form a close upper bound as was expected. The results are far better than those based on Equation 6.1.



**Figure 6.8 Equivalent Capacity vs. Burst Length**



**Figure 6.9 Equivalent Capacity vs. No. of Sources**

The multiplexing effect is explicitly shown in Figure 6.9 by plotting the effective bandwidth of single source as the number of sources increase. While there is no decrease in effective bandwidth per source when using the additive bandwidth method of [GUE91] based on Equation 6.1, the effective bandwidth obtained using the approaches in Section 3.3.3 and 3.4 clearly demonstrate the multiplexing advantage. The sources used for this example have Peak Rate of 4Mbps, source utilization factor  $\rho=0.3$  and burst length of 0.1 sec.

## 6.6 Summary

A call admission control based on fluid flow model has been developed. The effective bandwidth obtained for multiplexed On-Off sources is very close to the exact asymptotic solution obtained by solving the matrix differential equations. The approach presented here is computationally simple and can be used for real time call admission management. This method is sensitive to accurate characterization of the source. Hence there is a need for some other metric which may be used along with the effective bandwidth for admitting or rejecting a connection. Such a metric should be such that online measurement of this metric should be possible. One such metric may be the mean duration for which a node remains in congested state. The ratio of the duration for which a node remains in congestion to the duration of no congestion may form an interesting design parameter for call admission control. In general the preventive mechanism based on Call Admission Control and Traffic Policing result in over allocation of resources and hence under-utilisation and are also sensitive to inaccuracies in modeling of the source traffic. Hence the need for some kind of reactive mechanism along with the preventive mechanism so that the QoS requirements can still be met even under inaccurate description of source traffic parameters.

# **Chapter 7**

## **Conclusion**

In this thesis we have provided an analytical framework for engineering of QoS management system in next generation of packet switched networks. The next generation packet switched networks are expected to support multimedia traffic that requires guaranteed QoS from the network. This is only possible by combining various mechanisms like Call Admission Control, Policing, Scheduling, Flow Control etc. to achieve the goal of providing guaranteed QoS. Most of the schemes proposed in the literature are based on one of these approaches and perform Admission Control based on either loss probability or delay guarantees. In this work we have addressed the issue of flow control of real-time traffic by restricting the real-time flow to its guaranteed bandwidth during the duration when a node is experiencing congestion. The modern media encoders like GSM-AMR, MPEG4 etc allow multirate coding of media. This transmission of media traffic at reduced rate, only results in perceived degradation in quality by user. To make it a QoS criterion for Call Admission Control, this work has modeled the congestion duration and underload period and obtained the means of these periods. This model has been further extended to obtain the overload and underload periods for nodes with feedback controlled sources. These models have been used to show the impact of variation of controlled rate on the overload and underload periods.

The sojourn time analysis has been used to obtain the probability density for the over load and underload periods of a multiplexer with On-Off sources. The results have been applied to study the effectiveness of backpressure mechanism in reducing the congestion duration and lossy periods. Three different models have been used. One, that uses simple rate regulation to discard excess traffic during congestion. Second model allows buffering of excess traffic at source and results in higher mean congestion duration. In third model, non-negligible propagation delay has been added to the second model and its impact on sojourn times into congested and uncongested duration has been studied. The intractability of the analytical solution has been handled by making suitable approximations and the results have been validated by comparison with simulation results. In the end we have also provided a simple method

to obtain a better approximation for the tail probability distribution for a multiplexer. This takes into account the multiplexing effect of multiple sources when computing the effective bandwidth of traffic passing through a statistical multiplexer.

Three feedback models have been considered. Each of these models is suitable for different types of traffic sources. While first model suits real-time multimedia traffic, second model is more applicable for data sources. Hence a mix of two kind of traffics with separate control on both will result in lossless support for data traffic while keeping the mean congestion duration low, so as to provide QoS guarantee in terms of bounded delays for real-time multimedia traffic. The feedback model provides what may be termed as guaranteed bandwidth service to each source. This guaranteed rate forms an important parameter in admitting a call as reduced guaranteed rate will either result in perceived degradation in quality for the duration for which a multiplexer remains in congestion or increased duration of congestion in the case of buffered sources.

This analytical framework has application in engineering of QoS guarantees for packet networks with a mix of real-time and non-real time traffic. The application of this framework is discussed next.

## 7.1 Application

An integrated framework for traffic management, using preventive and reactive measures, has been suggested by the author in [MUD95A]. The framework in [MUD95A] can be divided into three parts.

1. Preventive measure for congestion control in the form of Call Admission Control (CAC) to restrict the traffic at a switching node. Effective bandwidth method of [MUD95A] is used to accept a call assuming that traffic on each virtual circuit is of On-Off type with exponentially distributed On and Off periods.
2. Feedback based control of the rate of traffic arriving at the congested node aims at providing a minimum guaranteed bandwidth to each virtual circuit, till the congestion is removed.
3. Scheduling mechanism before and after the congestion assures bounded delay guarantees to real-time traffic while providing loss-less service to loss sensitive non-real time traffic.

In [MUD95A], the traffic on each virtual circuit has been assumed to be of On-Off type, same as that discussed in Section 3.1.1 of the thesis. The virtual circuits are classified into real time and non-real time circuits. QoS requirements of real time circuits are bounded delay and that of non-real time connections is low cell loss ratio and low average delay. It is assumed that real time traffic is less sensitive to cell loss than non-real time traffic.

The output port of a non-blocking switch is modeled as a statistical multiplexer with link of capacity  $C$  with virtual circuits from different incoming links being multiplexed on the output link. The multiplexer is said to be in congested state when the queue-length crosses the high threshold mark. This threshold is decided dynamically based on number of existing connections. The congested node sends three types of signals to the upstream nodes: *reduce*, *stay* and *free*.

Call Admission Control is performed using effective bandwidth method of Section 6.3. The asymptotic constant is computed by obtaining the probability of the instantaneous inflow rate of combined real time and non-real time traffic being greater than the output link capacity  $C$ . If the sum of effective bandwidths of the existing and new virtual circuits is less than the link capacity then the new connection request is accepted else rejected. Different buffer sizes are used for real time and non real time traffic since real time traffic requires bounded delay guarantees and hence the smaller buffer.

On detecting the congestion, the congested node goes into controlled mode and takes the following sequence of actions:

1. Send *decrease* signal to all upstream nodes and reset *stay\_flag*
2. Start controlled mode scheduling mechanism
3. Wait for max round trip propagation delay
4. Monitor queue length until congestion ends
  - a. If non-real time queue length increases and *stay\_flag* is not set, then issue stay signal and set *stay\_flag*
  - b. If queue length increases and *stay\_flag* is set then goto step 1.
  - c. If queue length reduces below low threshold, send *free* signal, stop monitoring of queue length, reset *stay\_flag* and goto 5.
5. Start scheduling mechanism for open loop mode.

Every upstream node on receiving a *decrease* signal from congested downstream node, takes following set of actions while continuing to operate in open loop mode until it faces congestion due to backpressure mechanism:

1. Decrease the output cell rate to congested node to sum of equivalent capacities of the virtual circuits common to the two nodes, and strictly policing it.
2. Start increasing the output traffic to the congested node at a predefined rate. This rate is determined based on number of virtual circuits at the congested node and the difference in peak rates and effective bandwidth of each virtual circuit on congested node [MUD95A].
3. On receipt of *stay* signal, reduce the output cell rate to the rate existing at a time instant of round trip duration before receiving the *stay* signal.
4. Continue till *decrease* or *free* signal is received. If *decrease* signal is received then goto Step 1 else if *free* signal then stop policing the output traffic to the congested node.

Different scheduling strategy is used in open-loop mode and congested state. In open-loop mode simple non-preemptive priority is given to real time traffic. In congested state, the output link bandwidth is partitioned between real time traffic and non real time traffic. This partitioning is dynamically controlled to ensure zero loss in non-real time traffic while simultaneously providing maximum possible bandwidth to real time traffic.

The framework described above has been studied in [MUD95A] using simulation. It was observed that reactive feedback mechanism not only reduces the probability of sustained congestion but also assures that the contracted QoS is satisfied even when the network is heavily loaded. But this framework does not take into account the duration for which the node remains in congested state resulting in the degradation in the quality observed by the receiver of real-time traffic at reduced bit rate. Hence the ratio of mean congested and open-loop periods and the mean duration of congestion can supplement the Call Admission Strategy. This is discussed next.

Consider the two queue model described above. Let the real-time buffer be of size zero. While the feedback control of non-real time traffic follows the Model 2 described in Chapter 5, the feedback control associated with real-time traffic

is of Model 1 type. The scheduler on the output link uses following scheduling strategy:

- It gives priority to real time traffic when the multiplexer is in uncongested state
- In congested state, it gives only guaranteed bandwidth to the real-time traffic and rest of the bandwidth of output link is given to non-real time traffic

The analytical framework developed in this thesis has utility in engineering of Call Admission Control at each of such element in network backbone. Let the output link capacity be  $C$  and the combined flow of real-time traffic has following parameters:

- Peak Traffic  $R_p$
- Mean Traffic  $R_m$
- Guaranteed Traffic  $R_g$

Assuming that the traffic parameters satisfy the following relationship:

$$R_p > R_g > R_m \quad \text{----(7.1)}$$

The scheduling policy at the output of the multiplexer assures that the mean bandwidth available for the non-real-time traffic is greater than  $C_{NRT}$  where  $C_{NRT}$  is given by

$$C_{NRT} = C - R_g \quad \text{----(7.2)}$$

Such a system can be designed through proper engineering of high and low thresholds [MUD95A], [MUD98] to provide almost negligible loss probability. For such a system the mean delay experienced by non-real-time traffic will be an important criterion for Call Admission Control. For engineering purposes, the mean delay is obtained by using the approximate buffer occupancy distribution obtained in Chapter 6. This mean delay is used to obtain the effective bandwidth for the non-real-time traffic. The approach given in Section 6.3 is used for this. The asymptotic constant is obtained by computing the probability of instantaneous input rate being greater than the total output link capacity  $C$ . If  $C_{eq}$  is the effective bandwidth of the non-real time traffic so that mean delay is less than the specified mean delay then the following criterion need to be satisfied for Call Admission.

$$R_p < C \quad \text{----(7.3)}$$

$$C_{eq} < C_{NRT} \quad \text{----(7.4)}$$

Supplement these conditions with the check on if mean overload period and ratio of mean overload period to mean underload period satisfy the QoS specification before accepting a new connection.

Hence the analytical framework developed in this thesis has its usefulness in engineering of Call Admission Control to provide guaranteed Quality of Service for multimedia traffic and also the non-real-time data traffic in next generation packet switched networks.

## 7.2 Summary of the Thesis

In Chapter 3, the differential equation in Laplace domain has been obtained for probability density function of first passage to congestion and underload states of a multiplexer. The boundary conditions for the first passage times to the two states have been defined. Explicit expressions for eigenvalues and eigenvectors of the key matrix in the matrix differential equation for the first passage time probability density function, have been obtained. The properties of eigenvalues have been discussed. These properties are used along with the boundary conditions to obtain the solution for first passage density.

Method for obtaining the solution of first passage density function is provided in Chapter 4. The constant coefficients in the solution for first passage time to overload and underload states have been obtained for both finite buffer and infinite buffer models of a multiplexer. The Chapter describes a fast simulator developed as part of this work and discusses examples to show the sojourn time behavior of overload and underload states as high and low thresholds are varied. The mean overload period shows a linear dependence on difference between high and low thresholds and increases as this difference is increased. Mean underload period increases with increase in high or low threshold. This increase is of exponential nature.

In Chapter 5, the results obtained from earlier Chapters are applied to multiplexer with feedback controlled sources. As discussed earlier in this Chapter, three models have been used. Problem of modeling sojourn times of a multiplexer with feedback controlled sources with and without source buffer, has been made analytically tractable by approximating the model. These approximations are discussed in details in this Chapter. Results show the effectiveness of reactive mechanism in reducing congestion duration. Addition of source buffer increases

congestion duration but reduction in loss probability is a significant gain. For model with negligible propagation delay, it is shown that lossless transmission is possible. This study gives insight into the behavior of congestion duration and congestion cycle as feedback is applied. The first model shows significant reduction in congestion duration with feedback control while in second model feedback control results in lossless transmission of data but increases the duration for which multiplexer remains in overload state.

Call Admission Control algorithm based on Effective Bandwidth method has been described in Chapter 6. The solution uses different approach to obtain stationary probability and the structure is exploited to obtain the multiplexing gain. CAC algorithm is proposed which incorporates this multiplexing gain to obtain effective bandwidth for existing and new connection.

### **7.3 Scope of Future Work**

This study of feedback model uses multiplexer with constant rate output. This model developed here may be extended to multiplexer with phase type output process. This phase type output will be able to incorporate output control due to feedback from downstream node. The analysis of such a model will lead to more realistic modeling of backpressure mechanism.

Framework developed here has not been extended to obtain Call Admission Control strategy incorporating sojourn time in overload and underload study. The results obtained in this work may be extended to develop design rules for a network node, in terms of dynamic adjustment of high and low thresholds to provide required QoS to existing connections.

The effective bandwidth approach to CAC may be extended based on equivalent source concept. Let there be a simple source, which results in same moments of overload and underload periods as those by superposition of number of heterogeneous sources. The effective bandwidth of this source may then be used in CAC to admit new connections. Such a scheme will be useful for obtaining equivalent source from measured overload and underload periods in a multiplexer node.

# Appendix 1

## Properties of Eigenvalues

The procedure used to obtain the properties of the eigenvalues given by Equation 3.33 is discussed here. The eigenvalues given by Equation 3.33 are either positive eigenvalues with positive real part or are negative with negative real part. In the procedure used here the properties of eigenvalues are observed in right half of s-plane by obtaining the characteristics of the eigenvalues at  $s=0$  and how it changes in the positive half of s-plane in terms of its gradient. This approach is used to segregate the positive eigenvalues from the negative eigenvalues.

For  $s=0$  the coefficients  $a(l,s)$ ,  $b(l,s)$  and  $c(l,s)$  are given by:

$$\begin{aligned} a(l,0) &= \left(\frac{N}{2} - l\right)^2 R^2 - \left(\frac{N}{2} R - C\right)^2 \\ b(l,0) &= -2(\mu - \lambda)\left(\frac{N}{2} - l\right)^2 R + 2\left(\frac{N}{2} R - C\right)\left[\frac{N}{2}(\mu + \lambda)\right] \\ c(l,0) &= (\mu + \lambda)^2 \left[\left(\frac{N}{2} - l\right)^2 - \left(\frac{N}{2}\right)^2\right] \end{aligned} \quad (\text{A1.1})$$

For the given value of the parameters of the homogeneous sources and the link capacity of the multiplexer with fluid buffer, the two parameters which affect the sign of the coefficients  $a(l)$ ,  $b(l)$  and  $c(l)$  are the integer  $l$  and link capacity  $C$ . The parameter  $s$  is ignored when referring to the coefficients  $a(l,s)$ ,  $b(l,s)$  and  $c(l,s)$  for the case  $s=0$ . The properties of the eigenvalues  $z$  in the different region of  $C$  as the integer  $l$  vary from 0 to  $N$  are discussed next. The assumption of stability for the fluid multiplexer requires  $C$  to satisfy the following equation.

$$C > NR \left( \frac{\lambda}{\lambda + \mu} \right) \quad (\text{A1.2})$$

Consider the region  $C < NR/2$ . The root of the quadratic  $a(l)z^2 + b(l)z + c = 0$  is given by Equation 3.33. As  $l$  varies from 0 to  $N$  in Equation 3.33, it accounts for two roots due to the  $\pm$  sign of the term  $\sqrt{b^2 - 4ac}/2a$ . For  $l = 0$ , the coefficient  $c(l) = 0$  and  $a(l) > 0$ . Hence the root given by

$$z = -\frac{b(0)}{2a(0)} + \frac{\sqrt{b^2(0)}}{2a(0)} \quad (\text{A1.3})$$

depends on the sign of  $b(0)$ . Simplifying the expression for  $b(l)$ , with  $l = 0$ , we get

$$b(0) = N^2 R \lambda - NC(\lambda + \mu) \quad (\text{A1.4})$$

Assuming that the multiplexer satisfies the stability condition in Equation A1.2 then  $b(0)$  is negative. Thus for  $l = 0$ , the roots of the quadratic equations are non-negative and is given by

$$z = \left| \frac{b(0)}{a(0)} \right| \quad (\text{A1.5})$$

The first derivative of  $z(l,s)$  with respect to  $s$  is obtained next. The characteristics of the first derivative define how the root will behave in right half of  $s$ -plane.

$$\frac{dz(l,s)}{ds} = -\frac{\left(\frac{N}{2}R - C\right) - \left(\frac{N}{2} - l\right)R}{(NR - lR - C)(C - lR)} \frac{[sR + \mu C + \lambda(NR - C)]}{\sqrt{(sR + \mu C + \lambda(NR - C))^2 + 4\mu\lambda(NR - lR - C)(lR - C)}} \quad (\text{A1.6})$$

For  $l = 0$

$$\frac{[sR + \mu C + \lambda(NR - C)]}{\sqrt{(sR + \mu C + \lambda(NR - C))^2 + 4\mu\lambda(NR - lR - C)(lR - C)}} \geq 1 \quad \text{for all } s \geq 0 \quad (\text{A1.7})$$

Hence

$$\frac{dz(l,s)}{ds} \geq \frac{\left(\frac{N}{2}\right)R - \left(\frac{N}{2}R - C\right)}{(NR - C)C} > 0 \quad (\text{A1.8})$$

Therefore the non-negative eigenvalue for  $l = 0$  at  $s = 0$  remains non-negative for  $s \geq 0$  i.e. in the right half of  $s$ -plane.

Now consider  $0 < l \leq \lfloor C/R \rfloor$ . Once again from the definition of the coefficients of the quadratic equation it is observed that  $c(l) < 0$  and  $a(l) > 0$ . This implies that the sign of the root is determined by the sign of the third term in Equation 3.33. The numerator of this third term is positive and  $(NR - lR - C) > 0$  and  $(C - lR) > 0$  therefore the corresponding root and hence the corresponding eigenvalue is non-negative. To check the behavior of the root in the right half of  $s$ -plane the second term in Equation A1.6 satisfies the following inequality

$$\frac{[sR + \mu C + \lambda(NR - C)]}{\sqrt{(sR + \mu C + \lambda(NR - C))^2 + 4\mu\lambda(NR - lR - C)(lR - C)}} \geq 1 \quad \text{for all } s \geq 0 \quad (\text{A1.9})$$

And the derivative satisfies the following:

$$\frac{dz(l, s)}{ds} \geq \frac{\left(\frac{N}{2} - l\right)R - \left(\frac{N}{2}\right)R - C}{(NR - lR - C)(C - lR)} > 0 \quad (\text{A1.10})$$

Therefore it can be said that the eigenvalues corresponding to  $0 < l \leq \lfloor C/R \rfloor$  remain non-negative in the right half of s-plane.

Next consider the case  $\lfloor C/R \rfloor < l \leq N - \lfloor C/R \rfloor$ . The quadratics corresponding to the integer  $l$  in the range given have  $a(l) < 0$  and  $c(l) < 0$ . Hence the sign of  $b(l)$  will govern the properties of the root. Since  $C < NR/2$  and the system parameters satisfy the stability condition (Equation A1.2), the first term in Equation 3.33 is always positive as  $\mu > \lambda$ . The numerator of the second term is positive but the denominator is negative hence  $-b(l) > 0$ . Thus the roots corresponding to  $l$  in this range are also non-negative.

Also

$$\frac{[sR + \mu C + \lambda(NR - C)]}{\sqrt{(sR + \mu C + \lambda(NR - C))^2 + 4\mu\lambda(NR - lR - C)(lR - C)}} < 1 \quad \text{for all } s \geq 0 \quad (\text{A1.11})$$

and  $(NR - lR - C) > 0$  and  $(C - lR) < 0$ . This leads to following inequality:

$$\frac{dz(l, s)}{ds} \geq \frac{\left(\frac{N}{2}R - C\right) - \left(\frac{N}{2} - l\right)R}{(NR - lR - C)(lR - C)} > 0 \quad (\text{A1.12})$$

Hence the roots increase with increasing  $s$ . Thus it can be said that the roots remain non-negative for all  $s \geq 0$ .

For  $N - \lfloor C/R \rfloor < l \leq N$ , it is observed that  $a(l) > 0$  and  $c(l) < 0$ . Hence the properties of the roots are governed by the third term in Equation 3.33. This term is negative for the integer  $l$  such that  $N - \lfloor C/R \rfloor < l \leq N$ . Hence the corresponding eigenvalues are negative for  $s = 0$ . For  $l=N$  the root is zero.

$$\frac{[sR + \mu C + \lambda(NR - C)]}{\sqrt{(sR + \mu C + \lambda(NR - C))^2 + 4\mu\lambda(NR - lR - C)(lR - C)}} > 1 \quad \text{for all } s \geq 0 \quad (\text{A1.13})$$

Therefore

$$\frac{dz(l, s)}{ds} < \frac{\left(\frac{N}{2} - l\right)R - \left(\frac{N}{2}R - C\right)}{(NR - lR - C)(C - lR)} < 0 \quad (\text{A1.14})$$

Hence, the eigenvalues remains negative in the right half of s-plane.

Now consider  $C > NR/2$ . Following the approach taken above he properties of the eignevalues given by the set of quadratics defined by the constants given in Equation A1.1 are obtained. For  $l = 0$  the coefficients  $a(l) > 0$  and  $c(l) = 0$ . Hence the root of the quadratic is non-negative and is once again given by

$$z = \left| \frac{b(0)}{a(0)} \right| \quad (\text{A1.15})$$

Similarly for  $0 < l < N - \lfloor C/R \rfloor$   $a(l) > 0$  and  $c(l) < 0$ . Thus the roots for these values of l will depend on the properties of the third term in Equation 3.33. This term is non-negative implying the roots are non-negative for  $s = 0$ . Since  $(NR - lR - C) > 0$  and  $(lR - C) < 0$ , therefore

$$\frac{[sR + \mu C + \lambda(NR - C)]}{\sqrt{(sR + \mu C + \lambda(NR - C))^2 + 4\mu\lambda(NR - lR - C)(lR - C)}} > 1 \quad \text{for all } s \geq 0 \quad (\text{A1.16})$$

and

$$\frac{dz(l, s)}{ds} > \frac{\left(\frac{N}{2} - l\right)R - \left(\frac{N}{2}R - C\right)}{(NR - lR - C)(C - lR)} > 0 \quad (\text{A1.17})$$

For  $N - \lfloor C/R \rfloor \leq l \leq \lfloor C/R \rfloor$  it follows that  $a(l) < 0$  and  $c(l) < 0$ . Hence root  $z$  will depend on the sign contributed by the first two terms in Equation 3.33. Assuming that  $\lambda \geq \mu$  (stability condition (Equation A1.2)) then the root is negative for all values of  $l$  in the above given range. This assumption is based on the fact that the capacity  $C > NR/2$  and assuming the efficient utilization of resources the mean source utilization given by  $\lambda/(\lambda + \mu)$  is not less than  $\frac{1}{2}$ .

$$\frac{[sR + \mu C + \lambda(NR - C)]}{\sqrt{(sR + \mu C + \lambda(NR - C))^2 + 4\mu\lambda(NR - lR - C)(lR - C)}} < 1 \quad \text{for all } s \geq 0 \quad (\text{A1.18})$$

as  $(NR - lR - C) < 0$  and  $(lR - C) < 0$ . Hence

$$\frac{dz(l, s)}{ds} < \frac{\left(\frac{N}{2} - l\right)R - \left(\frac{N}{2}R - C\right)}{(NR - lR - C)(C - lR)} < 0 \quad (\text{A1.19})$$

Thus the roots remain non-negative in complete right half of the s-plane.

In the end checking the conditions for  $\lfloor C/R \rfloor < l \leq N$  we find that the coefficient  $a(l) > 0$  and  $c(l) < 0$ . Hence the roots once again depend on the third term in Equation 3.33. The denominator is non-negative as both the factors  $(NR - lR - C)$ , and  $(C - lR)$  are negative. The numerator is negative for integer  $l$  in the above range.

$$\frac{[sR + \mu C + \lambda(NR - C)]}{\sqrt{(sR + \mu C + \lambda(NR - C))^2 + 4\mu\lambda(NR - lR - C)(lR - C)}} > 1 \quad \text{for all } s \geq 0 \quad (\text{A1.20})$$

and

$$\frac{dz(l, s)}{ds} < \frac{\left(\frac{N}{2} - l\right)R - \left(\frac{N}{2}R - C\right)}{(NR - lR - C)(C - lR)} < 0 \quad (\text{A1.21})$$

Hence the roots remain negative in the entire right half of s-plane.

# Appendix 2

## Left and Right Eigenvectors

In order to obtain the right eigenvectors from the left eigenvectors given by Equation 3.34, the procedure outlined here has been used. There is a diagonal matrix  $\tau$  that symmetrizes the transition rate matrix  $M$  of the modulating process  $M(t)$ . The diagonal matrix  $\tau$  can also symmetrize the key matrix  $D^{-1}(sI - M)$ . For the Birth and Death process defined in Section 3.3.1, the  $i^{\text{th}}$  diagonal element  $\tau_i$  of  $\tau$ , is given by

$$\tau_i = \sqrt{(\lambda / \mu)^i \binom{N}{i}} \quad (\text{A2.1})$$

If  $V$  and  $W$  are the right and left eigenvectors of the key matrix  $D^{-1}(sI - M)$ , and  $V'$  and  $W'$  are the right and left eigenvectors of the symmetric matrix  $\tau^{-1}D^{-1}(sI - M)\tau$  then

$$V' = \tau^{-1}V \quad (\text{A2.2})$$

and

$$W' = W\tau \quad (\text{A2.3})$$

Since  $\tau^{-1}D^{-1}(sI - M)\tau$  is symmetric hence  $V'$  and  $W'^T$  are identical. Using this in Equation A2.2 and Equation A2.3 it can be shown that the left and right eigenvectors  $W$  and  $V$  of  $D^{-1}(sI - M)$  have the following relationship in terms of transpose of  $W$

$$V = \tau^2 W^T \quad (\text{A2.4})$$

# Appendix 3

## Stationary Probability Distribution

In [ANI82], the authors have used the model similar to the one presented in the Section 3.1. An infinitely large fluid buffer is fed by  $N$  identical sources with exponentially distributed On-Off periods. The buffer is emptied by a constant capacity output link. The superposed arrival process is a fluid arrival process with the background process given by a birth-death process with  $N+1$  states. The stationary distribution of the content in the buffer is obtained using the spectral analysis of the key matrix  $D^{-1}M$  where  $D$  is the drift matrix given by

$$D = \text{diag}\{r_0-C, r_1-C, r_2-C, \dots, r_N-C\} \quad (\text{A3.1})$$

And  $M$  is the infinitesimal generator for the background birth and death process. Assuming that the stability condition is satisfied.

Let  $P_i(t, x)$  denote the probability that the source defined by the  $N+1$  state Markov Modulated Rate Process (MMRP) is in state  $i$  and the buffer content does not exceed  $x$ .

$$P_i(t, x) \equiv P[M(t) = i, X(t) \leq x] \quad t \geq 0, x \geq 0, i \in [0, N] \quad (\text{A3.2})$$

then the Markov Process  $(M(t), X(t))$  is defined by the following differential equation obtained using Kolmogorov forward equations

For  $i \in [0, N]$

$$P_i(t + \Delta t, x) = \lambda_i \Delta t P_{i-1}(t, x) + \mu_i \Delta t P_{i+1}(t, x) + [1 - \mu_i \Delta t - \lambda_i \Delta t] P_i(t, x - (r_i - C) \Delta t) + o(\Delta t^2) \quad (\text{A3.3})$$

This results in the following set of partial differential equations as  $\Delta t \rightarrow 0$

$$\frac{\partial P_i(t, x)}{\partial t} + (r_i - C) \frac{\partial P_i(t, x)}{\partial x} = -(\lambda_i + \mu_i) P_i(t, x) + \lambda_{i-1} P_{i-1}(t, x) + \mu_{i+1} P_{i+1}(t, x) \quad i \in [0, N] \quad (\text{A3.4})$$

The stationary behavior of the system is given by time-independent equilibrium probabilities

$$F_i(x) = \lim_{t \rightarrow \infty} P_i(t, x) \quad i \in [0, N] \quad (\text{A3.5})$$

Thus

$$\frac{\partial P_i(t, x)}{\partial t} \equiv 0 \quad (\text{A3.6})$$

hence the system equilibrium probabilities are given by the following set of differential equations

$$(r_i - C) \frac{\partial F_i(x)}{\partial x} = -(\lambda_i + \mu_i) F_i(x) + \lambda_{i-1} F_{i-1}(x) + \mu_{i+1} F_{i+1}(x) \quad i \in [0, N] \quad (\text{A3.7})$$

This is given in matrix differential form as:

$$D \frac{dF(x)}{dx} = MF(x) \quad x \geq 0 \quad (\text{A3.8})$$

where  $D$  and  $M$  are  $(N+1) \times (N+1)$  matrix and  $F(x)$  is column vector as given below

$$D \equiv \begin{bmatrix} r_0 - C & 0 & . & . & 0 \\ 0 & r_1 - C & 0 & . & 0 \\ . & . & . & . & . \\ 0 & . & . & . & 0 \\ 0 & 0 & 0 & 0 & r_N - C \end{bmatrix} \quad (\text{A3.9})$$

$$M \equiv \begin{bmatrix} -\lambda_0 & \mu_1 & 0 & . & 0 \\ \lambda_0 & -\lambda_1 - \mu_1 & \mu_2 & . & 0 \\ . & . & . & . & . \\ 0 & . & . & . & 0 \\ 0 & 0 & 0 & \lambda_{N-1} & -\mu_N \end{bmatrix} \quad (\text{A3.10})$$

$$F(x) = [F_0(x) \ F_1(x) \ . \ . \ F_N(x)]^T \quad x \geq 0 \quad (\text{A3.11})$$

This system of equation has been solved in [ANI82] by obtaining the eigenvalues and eigenvectors for the matrix  $D^{-1}M$  and using the property of the solution at the boundary  $x=0$  and as  $x \rightarrow \infty$ . If the drift  $D$  is non-singular and  $M$  is irreducible then the solution of (A3.8) is given by

$$F_i(x) = F_i(\infty) - \sum_{j=0}^N a_j e^{b_j x} \quad i \in [0, N] \quad (\text{A3.12})$$

The number of eigenvalues with negative real part correspond to the number of states for which the drift  $d_i$  is positive. Similarly the number of eigenvalues with positive real part equals the number of states for which drift  $d_i$  is negative. The boundary value

requirement as  $x \rightarrow \infty$  results in the coefficients  $a_y = 0$  for all eigenvalues  $b_j$  which have positive real part. Hence if  $N_1$  is the number of states with the positive drift then

$$F_i(x) = F_i(\infty) - \sum_{j=0}^{N_1} a_j e^{b_j x} \quad i \in [0, N] \quad (\text{A3.13})$$

The stationary loss probability is obtained from the probability of the buffer content exceeding the buffer size  $B$  in (A3.13) for large values of  $x$ . For computing the effective bandwidth, the asymptotic loss probability is used. The advantage of asymptotic results is that they are based on dominant eigenvalue and the analytical formulas are simple, hence more suitable for implementing real time Call Admission Control.

For the arrival from a single On-Off source ( $N=1$ ) with peak rate  $R$  and the buffer link capacity  $C$ , the stationary probability distribution for the content in the fluid buffer is given by

$$F(x) = 1 - \frac{\lambda R}{(\lambda + \mu)C} e^{-x((\lambda + \mu)C - \lambda R)/C(R-C)} \quad x \geq 0 \quad (\text{A3.14})$$

$$P_{loss} = \frac{\lambda R}{(\lambda + \mu)C} e^{-x((\lambda + \mu)C - \lambda R)/C(R-C)} \quad x \geq 0 \quad (\text{A3.15})$$

For the system where the combined arrival process is superposition of  $N$  homogeneous On-Off sources with peak rate  $R$ , the parameters of drift matrix  $D$  and infinitesimal generator  $M$  are given by:

For  $i \in [0, N]$

$$\begin{aligned} r_i &= iR \\ \lambda_i &= (N - i)\lambda \\ \mu_i &= i\mu \end{aligned} \quad (\text{A3.16})$$

The asymptotic overflow probability for the infinite fluid buffer with output link capacity  $C$  has been obtained in [ANI82]

$$P_{loss} \sim \left( \frac{\lambda}{\lambda + \mu} \right)^N \left\{ \prod_{j=1}^{N_1} \frac{b_j}{b_j + b_N} \right\} e^{b_N x} \quad (\text{A3.17})$$

where  $b_j$  are the eigenvalues of the key matrix  $D^{-1}M$  and  $b_N$  is the dominant eigenvalue given by

$$b_N = -\frac{N((\mu + \lambda)C - N\lambda R)}{C(NR - C)} \quad (\text{A3.18})$$

This requires obtaining all the eigenvalues of the matrix  $D^{-1}M$ . In the literature on Call Admission Control based on effective bandwidth method [GUE91] [GIB95] [ELW93], the following approximation for loss probability has been used.

$$P_{loss} \sim e^{bx} \quad \text{for large } x \quad (\text{A3.19})$$

where  $b$  is the dominant eigenvalue. This is because of computation difficulty associated with computing the asymptotic constant  $A$  in the asymptotic overflow probability:

$$P(X > x) = Ae^{bx} \quad (\text{A3.20})$$

As shown in [ANI82] [KOS74] the constant  $b$  can be obtained from relatively simple expressions, while the determination of the constant  $A$  is typically more difficult [GUE91]. Approximating  $A$  by unity results in overestimation of the effective bandwidth as it does not take into account the multiplexing effect of the number of sources.

## Appendix 4

### Two-State MMRP Approximation

The superposition of  $N$  On-Off sources is approximated by a two state Markov Modulated Rate Process by using the approximation based on asymptotic matching [BAI91]. Let  $M$  be the modulating Markov Chain for the superposed traffic from  $N$  On-Off sources. The phases of the superposed arrival is divided in two based on positive and negative drift at the multiplexer. State 1 of the two-state MMRP corresponds to phases of superposed traffic that result in positive drift and state 0 represents phases with negative drift. Assuming  $R$  to be the peak rate of each contributing On-Off Source and  $C$  be the link capacity of multiplexer.

Let

$\lambda$  Mean transition rate of 2-state MMRP to state 1

$\mu$  Mean transition rate of 2-state MMRP to state 0

$r_1$  Fluid rate in state 1 of MMRP

$r_0$  Fluid rate in state 0 of MMRP

If  $Q$  is the  $[N - \lfloor C/R \rfloor] \times [N - \lfloor C/R \rfloor]$  rate transition matrix relevant to the transient states consisting of all the states of  $M$  that contribute to positive drift. Then  $\mu$  is given by the maximal real part eigenvalue of  $Q$ .

If  $\pi_i$  is the stationary probability of MMRP  $M$  being in state  $i$  then

$$r_0 = \sum_{i=0}^{\lfloor C/R \rfloor} \frac{\pi_i}{\sum_{j=0}^{\lfloor C/R \rfloor} \pi_j} iR \quad (\text{A4.1})$$

$$r_1 = \sum_{i=\lfloor C/R \rfloor + 1}^N \frac{\pi_i}{\sum_{j=\lfloor C/R \rfloor + 1}^N \pi_j} iR \quad (\text{A4.2})$$

$\lambda$  is obtained by matching the overall mean rate of the superposed traffic with that of two state MMRP.

$$\lambda = \mu \cdot \frac{N\rho R - r_0}{r_1 - N\rho R} \quad (\text{A4.3})$$

where  $\rho$  is the source utilisation of an On-Off Source.

## Appendix 5

### Transient Solution for ATM Multiplexer

The transient solution for an ATM Multiplexer is derived in [TAN95] [Ren95]. If  $M$  is the transition rate matrix of the two-state MMRP considered in Section 4.2 and  $D$  is the drift, then Laplace of transient probability distribution  $F_X(x, t, x_0)$  is given by

$$\begin{aligned}\tilde{F}_X(x, s, x_0) &= F_X(x, 0)(sI - M)^{-1} \left\{ I - e^{-(sI - M)D^{-1}(x - x_0)} \right\} \\ &\quad + A(s)e^{-(sI - M)D^{-1}x}\end{aligned}\tag{A5.1}$$

where  $A(s)$  is a constant matrix obtained from the following boundary conditions:

1. For infinite buffer, the terms with negative eigenvalues are zero.
2. When incoming traffic is greater than the outgoing traffic, the probability of buffer being empty is zero.

## Bibliography

- [ABA92] Abate, J., Whitt, W., "The Fourier Series method for inverting transforms of probability distributions", Queueing Systems, vol. 19, 1992, pg. 5-88
- [ALT] Altman, E., Baccelli, F., Bolot, J.C., "Discrete-Time analysis of Adaptive Rate Control Mechanisms", available on Internet
- [ANI82] Anick. D., Mitra, D. and Sondhi, M. M., "Stochastic theory of a data handling system with multiple sources", BSTJ Vol. 61, No. 8, Oct. 1982, pg. 1871-1894
- [ASM91] Asmussen, S., "Ladder heights and the Markov modulated M/G/1 queue", Stochastic Process Application, vol. 37, 1991, 313-326
- [ATL00] Atlasis, A.F., "The use of learning algorithm in ATM networks Call Admission Control problem: a methodology", Computer Networks, vol. 34, No. 3, 2000, pg. 341
- [ATM99] ATM Forum Traffic Management Specification, vers. 4.1, March 1999.
- [BAI91] Baiocchi, A, et. Al., "Buffer dimensioning criteria for an ATM multiplexer loaded with homogenous ON-OFF Sources", ITC 1991.
- [BAI91] Baiocchi, A. et al., "Loss performance analysis of an ATM multiplexer loaded with high speed On-Off sources", IEEE Journal on Selected Areas in Communication vol. 9, No. 3, April 1991, pg. 388-393
- [BAI92] Baiocchi, A., "Asymptotic behavior of the loss probability of the M/G/1/K and GI/M/1/K queues", Queueing Systems, vol. 10, 1992, pg. 235-248
- [BAI93] Baiocchi, A., "Accurate formulae for the loss probability evaluation in a large class of queuing system", Performance Evaluation, vol. 18, 1993, pg. 125-132
- [BAL90] Bala, K., Cider, I., Sohraby, K., "Congestion Control for High Speed Packet Switched Networks", Proceedings of IEEE INFOCOM '90, 1990, pg. 520-526
- [BEL76] Bellman, R.E. and Wing, "Introduction to Invariant Imbedding", Wiley, New York, 1976.
- [BEN63] Benes, V.E., "General Stochastic Processes in the Theory of Queues", Addison Wesley, 1963

- [BEN90] Bensaou, B., Gilbert, J., and Roberts, J.W., "Fluid queueing models for a superposition of on/off sources", Proc. ITC specialist seminar, Morristown, Oct. 1990, paper 9.3.
- [BEN98] Benmohamed, L., Wang, Y.T., "A control theoretic ABR explicit rate algorithm for ATM switches with per VC queuing", Proceedings of the IEEE INFOCOM '98, pg. 183-191
- [BER87] Bertsekas, D., Gallager, R., "Data Networks", Prentice Hall of India, 1987.
- [BER95] Beran, J., Sherman, R., taqqu, M.S., and Willinger, W., "Long-range dependence in variable-bit-rate video traffic", IEEE Trans. on Communications, vol. 43, 1995, pg. 1566-1579.
- [BLA98] Blake, S., et. al. , "An Architecture for Differentiated Services", RFC 2475, Dec. 1998.
- [BLO89] Blondia, C. "The N/G/1 finite capacity queue", Commun-stat-stocahstic models, vol. 5, No. 2, 1989, pg. 273-294
- [BLO92] Blondia, C., Casala, O., "Statistical Multiplexing of VBR sources: A matrix-analytic approach", Performance Evaluation, vol. 16, No-1-3, 1992, pg. 5-20
- [BOL91] Bolotin, V. A., Kappel, J.G. and Kuehn, P.J., "Teletraffic Analysis of ATM Systems", IEEE Journal on Selected Areas in Communications, vol. 3, No. 3, 1991.
- [BON95] Bonomi, F., Fendick, K., "The rate-based flow control framework for the available bit rate ATM service", IEEE Network, March 1995, pg. 25-39
- [BOT95] Botvich, D., Duffield, N., "Large deviations, the shape of the loss curve, and economics of scale in large multiplexers", Queueing Systems, vol. 20, 1995, pg. 293-320
- [BOU91] Boudec, J. T., "An efficient solution method for Markov Models of ATM Links with Loss Priorities", Proceedings of IEEE Infocom, 1991
- [BOY00] Boyle, J., et. al., "COPS usage for RSVP", RFC 2749, Jan 2000.
- [BRA69] Brady, P. T., "A Model for Generating On/Off Speech Patterns in two way Conversations", Bell Systems Technical Journal, Vol. 48, 1969, pg. 2445-2472
- [BRA94] Braden, R., Clarke, D., Shenker, S., "Integrated Service in the Internet Architecture: an Overview", RFC 1633, June 1994.

- [BRA97] Braden, R., et. al. , “Resource Reservation Protocol (RSVP) version 1: Functional Specification”, RFC 2205, Sept. 1997.
- [BUR98] Burst, S.C., Mitra, D., “Virtual partitioning for robust resource sharing: computational techniques for heterogeneous traffic”, IEEE Journal on Selected Areas in Communications, vol. 16, No. 5, 1998, pg. 668-678
- [BUT91] Butto, M., Cavallero, E., Tonietti, A., “Effectiveness of the “Leaky Bucket” Policing Mechanism in ATM Networks”, IEEE Journal on Selected Areas in Communication, vol .9, 1991, pg. 335-341
- [CHA94] Chang, C., “Stability, queue length, and delay of deterministic and stochastic queueing networks”, IEEE Transactions on Automatic Control, Vol. 61, 1982, pg. 1871-1894
- [CHA95] Chang, C.S., Thomas, J.A., “Effective bandwidth in high speed digital network”, IEEE Journal on Selected Areas in Communications, vol. 13, 1995, pg. 1091-1100
- [CHA98] Chan, Tat-Keung and Victor, O.K. Li, “Decomposition of Network of Queues with self-similar traffic”, Proceedings of IEEE Globecom, 1998, pg. 3001-3006
- [CHE90] Chen, J.S.-C. R. Guerin, Stern, T.E., “Markov modulated flow models for the output queues of a packet switch”, IEEE INFOCOM '90, 1990, pg. 719-727
- [CHE93] Xiaoqiang Chen, “Modeling Connection Admission Control”, Proceedings of INFOCOM '93, 1993, pg. 274-281.
- [CHE96] Chen, T.M., Lin, S.S., Samalam, V.K., “The available bit rate service for data in ATM networks”, IEEE Communication Magazine, May 1996, pg. 56-71
- [CHO] Choudhury, G.L., Lucantoni, D.M., and Whitt, W., “Tail probabilities in queue with many independent sources”, available on Internet
- [CHO1] Choudhury, G.L., Lucanotoni, D.M., Whitt, W., “Multidimensional transform inversion with applications to the transient M/G/1 queue”, available on Internet
- [CHO96] Choudhury, G., Lucantoni, D., Whitt, W., “Squeezing the most out of ATM”, IEEE Transactions on Communications, vol. 44 No. 2, 1996, pg. 203-217
- [CHO97] Choudhury, G.L., Mandelbaum, A., Reiman, M.Z., Whitt, W., "Fluid diffusion limits for queues in slowly changing environment", Stochastic Models, Vol. 13, No. 1, 1997, pg. 121-144

- [CHO98] Choe, J., Shroff, N., “A central limit theorem based approach to analyze queue behavior in ATM networks”, IEEE/ACM Transactions on Networking, vol. 6, No. 5, 1998, pg. 659-671
- [CLA88] Clarke, D., “The Design Philosophy of the DARPA Internet Protocol”, ACM SIGCOMM ‘88, August 1988.
- [CLA92] Clarke, D., Shenker, S., Zhang, L., “Supporting Real-Time Application in an Integrated Services Packet Network: Architecture and Mechanisms”, Proceedings of SIGCOMM ‘92, Baltimore, MD, August 1992.
- [DAI91] Daigle, J., and Langeford, J., “Models for Analysis of Packet Voice Systems”, IEEE Journal on Selected Areas of Communications, vol. 9, No. 3, April 1991, pg. 305-314.
- [DIG91] Dighe, R., “Congestion Avoidance Strategies in Broadband Packet Networks”, IEEE Infocom ‘91, 1991, pg. 295-303
- [DIN90] Ding, W., Decker, P., “Waiting time distribution of a discrete SSMP/G/1 queue and it’s implication in ATM system”, 7<sup>th</sup> ITC Special Seminar, Morristown, 1990
- [DUF95] Duffield, N. G., Neil O Connell, “Large deviations and overflow probabilities for the general single server queue with application”, Proceedings Cambridge Philos. Soc., 1995, pg. 363-374
- [DUF95A] Dufosfield, N. G., Lewes, J. J., et al, “Antropy of ATM Traffic Streams: A tool for Estimating QoS Parameters”, IEEE JSAC vol. 13, No. 6, Aug. 1995, pg. 981-990
- [DZI96] Dzhang, Z., Maran, L.G., “Fair-efficient call admission control policies for broadband networks- a game theoretic framework”, IEEE/ACM Transactions on Networking, vol. 4, No. 1, 1996, pg. 123-136
- [ECK79] Eckberg, A. E., “The single server queue with periodic arrival process and deterministic service time”, IEEE Transactions on Communications, Vol. 27, 1979, pg. 556-562
- [ELW93] Elwalid, A.I., and Mitra, D., “Effective bandwidth of general Markovian traffic sources and admission control of high speed networks”, IEEE/ACM Trans. Networking, vol. 1, No. 3, 1993, pg. 329-343
- [ELW95] Elwalid, A., Mitra, D., Wentworth, R., “A new approach for allocating buffers and bandwidth to heterogeneous regulated traffic in an ATM node”, IEEE Journal on Selected Areas in Communications, vol. 13, 1995, pg. 1115-1127
- [ELW95A] Elwalid, A., Heyman, D., Lakshman, T., Mitra, D., Weiss, A., “Fundamental bounds and approximations for ATM multiplexers with

- applications to video teleconferencing”, IEEE Journal on Selected Areas in Communications, vol. 13, No. 6, 1995, pg. 203-217
- [FEN] Fendick, K.W., Rodrigues, M.A., Weiss, A., “Analysis of rate-based feedback control strategy for long haul data transport”, Performance Evaluation
- [FEN92] Fendick, K.W., Rodrigues, M.A., Weiss, A., “Analysis of a rate based Control Strategy with Delayed feedback”, Proceedings of ACM SIGCOMM, 1992
- [FEN96] Fendick, K.W., “Evolution of controls for the available bit rate service”, IEEE Communication Magazine, Nov. 1996, pg. 35-39
- [FER90] Ferrari, D., Verma, D., “A scheme for real-time channel establishment in wide-area networks”, IEEE Journal on Selected Areas in Communications, Vol. 8, No. 3, 1990, pg. 368-379
- [FIS93] Fischer, W. and Kathleen Meier-Hellstern, “The Markov Modulated Poisson Process (MMPP) Cookbook”, Performance evaluation, vol. 18, 1993, pg 149-171
- [FLO92] Floyd, S., “Issue in Flexible Resource Management for Datagram Networks”, Proceedings of the 3<sup>rd</sup> workshop on very high Speed Networks, March 1992.
- [FLO00] Floyd, S., “Congestion Control Principles”, RFC 2914, Sept. 2000
- [FRI93] Friesen, V. J., Wong, J.W., “The Effect of Multiplexing, Switching and Other Factors on the Performance of Broadband Networks”, Proceedings of IEEE INFOCOM, 1993, pg. 1194-1203
- [GAR94] Garrett, M.W. and Willinger, W., “Analysis, modeling and generation of self-similar VBR video traffic”, Proceedings of ACM SigComm, 1994, pg. 269-280
- [GIB95] Gibbens, R., Kelly, F., Key, P., “A decision-theoretic approach to call admission control in ATM networks”, IEEE Journal on Selected Areas in Communications, vol. 13, No. 6, 1995, pg. 1101-1114
- [GLY94] Glynn, P., Whitt, W., “Logarithmic asymptotics for steady-state tail probabilities in a single-server queue”, Journal of Applied Probability”, 1994, pg. 131-155
- [GON86] Gonet, P., Adam, P., Coudrease, J.P., “Asynchronous Time Division Switching: The way to Flexible Broadband Communication Networks”, in Digital Communication-New Directions in Switching and Network, North Holland, IEEE, 1986.
- [GRA82] Grassman, W.K., “The GI/PH/1 queue: A Method to find the transition matrix”, Inform, vol. 20, 1982, pg. 144-156

- [GRA85] Grassman ,W., Taksar, M., Heyman, D., “Regenerative analysis and steady state distribution for Markov Chains”, Operations Research, Vol. 33, 1985, pg. 1107-1116
- [GRA89] Grassmann, W.K., Jain, J.L., “Numerical Solutions of the waiting Time and Idle Time Distributions of the Arithmatic GI/G/1 queues”, Operations Research, vol. 37, 1989, pg.141-150
- [GRO90] Grossglauser, M. and Bolot, J-C, “On the relevance of long-range dependence in network traffic”, in Proc. ACM SIGCOMM '96, 1996.
- [GRU82] Gruber, J.G., ‘A Comparison of Measured and Calculated Speech Temporal Parameters Relevant to Speech Activity Detection’, IEEE Trans. Communs., Vol. COM-30, No. 4, pp 728-738, 1982.
- [GRU83] Gruber, J. G., and Le, N. H., “Performance Requirements for Integrated Voice and Data Networks”, IEEE Journal on Selected Areas in Communication, Dec. 1983, pg. 981-1005
- [GOY99] Goyal, R, “Performance Analysis of TCP over ABR”, PhD Thesis, 1999
- [GUE91] Guerin, R. Ahmadi, H., and Naghshineh, M., “Equivalent capacity and its application to bandwidth allocation in high speed network”, IEEE Journal on Selected Areas in Communications, vol. 9, No. 7, 1991, pg. 968-981
- [GUS90] Gusella R. ,”A measurement study of Diskless Workstation Traffic on an Ethernet”, IEEE Trans. Communications, vol. 38, No. 9, 1990, pp 1557-1568.
- [GUS90] Gusella, R., “A measurement study fo Diskless Workstation Traffic on an Ethernet”, IEEE Trans. On Communications, vol. 38, No. 9, 1990, pg. 1557-1568
- [HAS91] Hashida, O. Y., Takahashi, Y., and Shimogawa, S., “Switched Batch Bernoulli Process (SBBP) and the Discrete-Time SBBP/G/1 Queue with Application to Statistical Multiplexer”, IEEE Journal on Selected Areas in Communications, vol. 9, No. 3, 1991, pg. 394-401
- [HEF80] Heffes, H., “A Class of Data Traffic processes”, Bell Systems Tech. Journal, vol. 59, 1980, pg. 897-929
- [HEF86] Heffes, H. and Lucantoni, D.M., “A Markov Modulated characterization of Packet Voice and Data traffic and Related Statistical Multiplexer Performance”, IEEE Journal on Selected Areas in Comm., No. 6, Sept. 1986, pg. 856-867

- [HER93] Herrman, C., "Analysis of the Discrete-time SMP<sup>[X]</sup>/D/1/s finite buffer queue with Application in ATM", Proceedings of IEEE INFOCOM, 1993
- [HEY92] Heyman, D.P., Lakshman, T.V., and Tabatabai, A., "Statistical analysis and simulation study of video teleconferencing traffic in ATM networks", IEEE Trans. Circuits Syst. Video Technology, vol. 32, 1992, pg. 49-59
- [HEY96] Heyman, D. P. and Lakshman, T. V., "What are the Implications of Long-Range Dependence for VBR-Video Traffic Engineering?", IEEE/ACM Trans. on Networking, vol. 4, no. 3, June 1996, pp. 301-17.
- [HOW00] How, Y.T., Li, B., Panwar, S.S., Tseng, H., "On network bandwidth allocation policies and feedback control algorithm for packet networks", Computer Networks, vol. 34, 2000, pg. 481-501
- [HSI93] Hsing, D.K., "Performance study on the "Leaky Bucket" Usage Parameter Control Mechanism with CLP tagging", Proceedings of IEEE GLOBECOM 1993, pg-359-364
- [HUE98] Huebnar, F., "On the Accuracy of Approximating Loss Probabilities in Finite Queues by probabilities to exceed queues levels in finite queues", Proceedings of IEEE GLOBECOM, 1998, pg. 484-489
- [HUI88] Hui, J. Y., "Resource allocation for broadband networks", Proceedings of ICC'88, Philadelphia, 1988, pg. 31-36
- [HYM93] Hyman, J.M., et. al. , "A Separation principle between scheduling and admission control for broadband switching", IEEE Journal on Selected Areas in Communications, vol. 11, No. 4, 1993, pg. 605-616
- [ITU96] ITU-T Recommendation I.371, "Traffic Control and Congestion Control in B-ISDN", May 1996
- [JAC] Jacobson, S.B, Diffman, L., "A Fluid Flow queueing model for heterogeneous On/Off traffic", on Internet
- [JAC87] Jacobson, V., "Congestion Avoidance and Control", Proceedings of the ACM SIGCOMM, Computer Communication Review, vol. 18, No. 4. Pg. 316-329
- [JAC57] Jackson, J. R., "Networks of Waiting Lines", Operations Research, 1957, pg. 518-521
- [JAI96] Jain, R., "Congestion control and traffic management in ATM Networks: recent advances and a survey", Computer Networks and ISDN system, vol. 28, 1996, pf. 1723-1738
- [JAM92] Jamin, S. et. al., "An Admission Control Algorithm for Predictive Real-Time Admission Control Algorithm for Predictive Real-Time Service", Time

- extended abstract in Proceedings of Third International Workshop on Network and Operating System support for Digital Audio and Video, San Diego, CA, Nov. 1992, pg. 73-91
- [JEL97] Jelenkovic, P., Lazar, A.A., and Semret, N., "The effect of multiple time scales and subexponentiality in MPEG video streams on queuing behavior", IEEE Journal on Selected Areas in Communications, vol 15, 1997, pg. 1052-1071.
- [KES91] Keshav, S. "A control-theoretic approach to flow control", Proceedings of ACM SIGCOMM '91
- [KES93] Kesidas, G., Walrand, J., Chang, C., "Effective bandwidths for multiclass Markov fluids and other ATM sources", IEEE/ACM Transactions on Networking", vol. 1, No. 4, 1993, pg. 424-428
- [KIM] Kim, H., Shroff, N., "Loss probability calculations in a finite buffer queueing system", Technical Report.
- [KIS89] Kishino, F., K. Manabe, Y. Hayashi, and H. Yasuda, "Variable Bit-Rate Coding of Video Signals for ATM Networks", IEEE Journal on Selected Areas of Communication, Vol. 7, No. 5, June 1989, pg. 801-806
- [KNI99] Knightly, E.W, Shroff, N.B., "Admission Control for Statistical QoS: Theory and Practice", IEEE Networks, vol. 13, No. 2, 1999, pg. 20-29
- [KOS74] Kosten, L., "Stochastic Theory of a Multi-Entry Buffer (1)", Delft Progress Report. Vol 1, 1974, pg. 10-18
- [KOS84] Kosten. L., "Stochastic theory of data-handling systems with groups of multiple sources", In Perfoemance of Computer-Communication Systems, 1984, pg. 321-331
- [KUC73] Kuczura, A., "The Interdeparture Poisson Process as an Overflow Process", Bell Systems Tech. Journal, vol. 52, 1973, pg. 473
- [LAB92] Labourdette, J. F. D., Hant, G. W., "Blocking probabilities in multitraffic loss systems: insensitivity, asymptotic behavior and approximations", IEEE Transactions on Communciations, vol. 40, No. 8, 1992, pg. 1355-1366
- [LAT93] Latouche, G. and Ramaswami, V., "A logarithmic reduction algorithm for quasi-birth-death process", Adv. Applied Probability, vol. 30, 1993, pg. 650-674
- [LAU93] Lau, W. and Li, San-qj, "Traffic Analysis in Large-Scale High Speed Integrated Networks: Validation of Nodal Decomposition Approach", Proceedings of IEEE Infocom, 1993, pg. 1320-1329
- [LAU00] Lau, W.C. and Li, S., "Sojourn-time analysis on nodal congestion in broadband networks", Computer Networks, vol. 32, 2000

- [LAU95] Lau, W.C., Erramilli, A., Wang, J.L., Willinger, W., "Self-Similar Traffic Generation: The Random Midpoint Displacement Algorithm and it's properties", ICC, 1995, pg. 466-472
- [LEE92] Lee, D.S. and Li, S.q., "Transient analysis of multi-server queues with Markov-Modulated Poisson arrivals and overload control", Performance Evaluation, vol. 16, 1992, pg. 49-66
- [LEE96] Lee, T, Lai, K., Duann, S., "Design of a real-time call admission controller for ATM networks", IEEE/ACM Transactions on Networking, vol 4. No. 5, 1996, pg. 758-765
- [LEL94] Leland, W. E., Willinger, W., et al , "On Self Similar Nature of Ethernet Traffic", IEEE/ACM Trans on Networking, vol. 2, No. 1, Feb. 1994, pg. 1-15
- [LI91] Li, Q.Q., Sheng, H.D., "Discrete queuing analysis of multi-media traffic with diversity of correlation and burstiness properties", IEEE INFOCOM, 1991, pg. 368-381
- [LI93] Li, S.Q., Hwang, C.", Queue Response to Input Correlation Function: Continuous Spectral Analysis", IEEE Trans. Networking, vol. 1, No. 6, 1993, pg. 678-692
- [LIE95] Liebehen, J., "Multimedia networks: issues and challenges", Computer vol. 28, No. 4, 1995, pg. 68-69
- [LIN98] George C. Lin and Tatsuya Suda, "On the Impact of Long-Range-Dependent Traffic in Dimensioning ATM Network Buffer", Proceedings of IEEE Infocom'98, March, 1998.
- [LUC] Lucantoni, A., Choudhury, G.L., Whitt, W., "The transient BMAP/G/1/Queue", <http://www.research.att.com/~wow/lucantoni.html>
- [LUC85] Lucantoni, D.M. and Ramaswamy, V.", Efficient algorithm for solving the non-linear matrix equation arising in phase type queues", Communication-Statistics and Stochastic Models, vol. 1, 1985, pg. 29-51
- [LUC91] Lucantoni, D.M., "New Results on a single server queue with a batch Markovian Arrival Process", Stochastic Models, vol. 7, 1991, pg-1-46
- [LUC93] Lucantoni, D., "New results on the single server queue with a batch Markovian arrival process", Stochastic Models, vol. 7., 1993, pg. 1-46
- [MAG88] Maglaris, B., Anastassiou, D., Sen, P., Karlsson, G. and Robins, J., "Performance Models for Statistical Multiplexing in Packet Video Communications", IEEE Journal on Selected Areas in Communications, Vol. 36, 1988, pg. 834-844

- [MEL98] Melamed, B., Pendarakis, D. E., "Modeling full-length vbr video using Markov-Renewal\_Modulated TES Models", IEEE Journal on Selected Areas in Communication, vol. 16, No. 5, June 1998, pg. 600-611
- [MIS92] Mishra, P.P., Kanakia, H., "A hop-by-hop rate based congestion control scheme", ACM Sigcomm '92, pg. 112-123
- [MIT96] Mitra, D., Morrison, J.A., Ramakrishnan, K.G., "ATM network design and optimization: a multirate loss network framework", IEEE/ACM Transaction on Networking, vol. 4, No. 4, 1996, pg. 531-547
- [MIT98] Mitra, D., Rieman, M.I., Wang, J., "Robust dynamic admission control for unified cell and call QoS in Statistical Multiplexers", IEEE Journal on Selected Areas in Communications, vol. 16, No. 5, 1998, 692-707
- [MUD95] Mudgil, V., Srivathsan, K.R., Bose, S.K., "Effective Bandwidth Method for Call Admission Control in ATM Network", Proceedings of the first National Conference on Communications, NCC'95, March 1995
- [MUD95A] Mudgil, V., Srivathsan, K.R., Bose, S.K., "Hop-By-Hop congestion control in ATM Networks, based on combined preventive and reactive mechanisms", Third Workshop on Performance Modeling and Evaluation of ATM Networks, 1995 Ilkley, West Yorkshire, U.K.
- [MUD97] Mudgil, V., Srivathsan, K.R., Bose, S.K., "A Fast Simulator Using Embedded Flow Equations for ATM Networks", Proceedings of the National Conference on Communication (NCC'97), I.I.T. Chennai.
- [MUK91] Mukherjee, A., Strikwerda, J.C., "Analysis of dynamic congestion control protocols-A fokker Planck approximation", Proceedings of ACM SIGCOMM, 1991, pg. 159-169
- [NAG91] Nagarajan, R., Kurose, J.F. and Towsley, D., "Approximation techniques for computing packet loss in finite buffered voice multiplexers", IEEE Journal on Selected Areas in Comm., vol. 9, No. 3, 1991, pg. 368-377
- [NAR98] Narayan, O., "Exact Asymptotic Queue Length Distribution for Fractional Brownian Motion Traffic", Advances in Performance Analysis, vol. 1, No. 1, 1998, pg. 39-63
- [NEU81] Neuts, M.F., "Matrix-Geometric Solutions in Stochastic Models: An Algorithmic approach", the John Hopkins University Press, 1981.
- [NOB58] Noble, B., "Methods based on the Wiener-Hopf Technique", Pergamon Press 1958
- [NOM89] Nomura, M., T. Fuji, and Ohta, N., "Basic Characteristics of Variable rate Video Coding in ATM environment", IEEE Journal on Selected Areas of Communication, Vol. 7, No. 5, June 1989, pg. 752-760

- [NOR91] Norros, I., Roberts, J.W., Simonian, A., Virtamo, J.T., "The superposition of variable bit rate sources in an ATM multiplexer", IEEE Journal on Selected Areas in Communications, vol. 9, No. 3, 1991, pg. 378-387
- [OHB91] Ohba, Y., Murata, M., Miyahara, H., "Analysis of Interdeparture Processes for Bursty Traffic in ATM Networks", IEEE Journal on Selected Areas in Communications, vol. 9, No. 3, 1991, pg. 468-476
- [ONV94] Onvural, R.O., "Asynchronous Transfer Mode Networks: Performance Issues", Artech House, 1994
- [PAX95] Paxson, V. and Floyd, S., "Wide-Area traffic: The failure of the Poisson modeling", IEEE/ACM Trans. Networking, 3(3) pg. 226-244, June 1995
- [PER96] Perros, H., Elsayed, K., "Call Admission Control schemes: A review", IEEE Communication Magazine, vol. 34, No. 11, 1996, pg. 82-91
- [PRA98] Prasad, S., Kiasalah, K., Balsair, P. "LAPLUS:an efficient, effective and stable switch algorithm for flow control of the available bit rate ATM service", Proceedings of the IEEE INFOCOM '98, 1998, pg. 174-182
- [PAZ95] Pazhyannur, R.S., Agrawal, R., "Feedback-Based Flow Control of B-ISDN/ATM Networks", IEEE Journal on Selected Areas in Communications, vol. 13, No. 7, 1995, pg. 1252-1266
- [RAM80] Ramaswamy, V., "The N/G/1 queue and its detailed analysis", Advanced Applied Probability, vol. 12, March 1980, pg. 222-261
- [RAM88] Ramaswamy, V., "Traffic performance modeling for packet communication: Whence, Where and Whither", Third Australian Teletraffic Seminar, Melbourne, Australia, 1988.
- [RAM88A] Ramakrishnan, K.K., Jain, R., "A binary feedback scheme for congestion avoidance in computer networks with a connectionless network layer", Proceedings of SIGCOMM, '88, pg. 303-313
- [RAM90] Ramaswamy, V., and Willinger, W.. "Efficient trafffic performance strategies for packet multiplexer", ISDN Networks and Systems, Dec. 1990, pg. 401-407
- [RAM91] Ramaswami, V., Rumscwicz, M., Willinger, W. and Eliazov, T., "Comparison of some traffic models for ATM performance studies", Proc. ITC-13, 1991, pg. 7-12.
- [RAM92] Ramamurthy, G. and Sengupta, B., "Modeling and analysis of a variable bit rate video multiplexor", Proceedings of IEEE INFOCOM '92, Florence, Italy, 1992, pg. 817-827

- [RAM93] Ramamurthy, S., Sengupta, D., “A predictive Hop-by-Hop Congestion Control policy for high speed networks” IEEE Infocom, 1993, pg. 1033, 1044
- [RAM97] Ramamurthy, G., Ren, Q., “Multiclass connection admission control policy for ATM switches”, in Proceedings of IEEE INFOCOM '97, 1997
- [RAT91] Rathgeb, E. B., “Modeling and Performance Comparison of Policing Mechanisms for ATM Networks”, IEEE Journal on Selected Areas in Communication, Vol. 9, No. 3, 1991, pg. 325-334
- [REN95] Ren, Q., Kobayashi, H., “Transient solutions for the buffer behavior in statistical multiplexing”, Performance Evaluation, vol. 29, 1995, pg. 65-87
- [REN00] Ren, Q., Ramamurthy, G., “A Real-Time Dynamic Connection Admission Controller based on Traffic Modeling, Measurement and Fuzzy Logic Control”, IEEE Journal on Selected Areas in Communications, Vol. 18, No. 2, 2000, pg. 184-196
- [ROB91] Roberts, J.W. and Virtamo, J.T., “The superposition of periodic cell arrival process in an ATM multiplexer”, IEEE Trans. Commun. Vol. 39, No. 2. 1991, pg. 298-303
- [RYU96] Ryu, B. and Elwalid, A. “The importance of long range dependence of VBR video traffic in ATM traffic engineering: Myths and realities”, in Proc. ACM SIGCOMM '96, 1996, pg. 3-14
- [SAH99] Sahinoglu, Z., and Tekinay, S., “On Multimedia Networks: Self-Similar Traffic and Network Performance”, IEEE Communication Magazine, January 1999.
- [SAI] Saito, H. et al, “An Analysis of Statistical Multiplexing in an ATM Transport Network”, available on Internet
- [SAI91] Saito, H., “Queuing analysis of cell loss probability control in ATM networks”, Proc. ITC-13, Copenhagen, June 1991, pg. 19-23
- [SAI91A] Saito, H., Kawarasaki, M., and Yamada, H., “An analysis of Statistical multiplexing in an ATM transport network”, IEEE Journal on Selected Areas in Communications, April 1991, pg. 359-367
- [SAI91B] Saito, H., Shimoto, K., “Dynamic call admission control in ATM networks”, IEEE Journal on Selected Areas in Communications, vol. 9, 1991, pg. 982-989
- [SAI93] Saito, H., “Teletraffic Technologies in ATM Networks”, Artech House Inc. 1993

- [SAI96] Saito, H. et al, "Performance Issues in Public ABR Service", IEEE Communications Magazine, Nov. 1996, pg. 40-48
- [SEN89] Sen, P., Maglaris, B., Rikli, N., and Anastassiou, D., "Models for Packet Switching of Variable Bit Rate Video Sources", IEEE Journal on Selected Areas of Communication, Vol. 7, No. 5, June 1989, pg. 865-869
- [SHA90] Saito, H., "The departure process of an N/G/1 queue", Performance Evaluation, vol. 11, 1990, pg. 241-251
- [SHA93] Shanker, S., Clarke, D. and Zhang, L., "A Scheduling Service Model and a Scheduling Architecture for an Integrated Services Packet Network", ACM/IEEE Transaction on Networking, 1993
- [SHI97] Shioda, S., Saito, H., "Real time cell loss ratio estimation and its application to ATM traffic controls", Proceedings of IEEE INFOCOM, 1997.
- [SHR98] Shroff, N., Schwartz, M., "Improved loss calculations at an ATM Multiplexer", IEEE/ACM Transactions on Networking, vol. 6, No. 4, 1998, pg. 411-422
- [SPE85] Speck, F.O., "General Wiener-Hopf factorization methods", Pitman Advanced Publishing Program, 1985
- [SRI86] Sriram, K., and Whitt, W., "Characterizing Superposition Arrival Processes in Packet Multiplexers for Voice and Data", IEEE Journal on Selected Areas in Communications, vol. 4, No. 6, 1986, pg. 833-846
- [STE91] Stern, T., Elwalid, A.I., "Analysis of Separable Markov-Modulated Rate Models for Information Handling Systems", Advanced Applied Probability, vol. 23, 1991, pg. 105-139
- [TAK62] Takacs, L., "Introduction to the Theory of Queues", New York, Oxford University Press, 1962
- [TAN95] Tanaka, T., Hashida, Q. and Takahashi, Y., "Transient analysis of fluid model for ATM Statistical multiplexer", Performance Evaluation, 1995, vol. 23, pg. 145-162
- [TAQ97] Taqqu, M.S., Willinger, W., and Sherman, R., "Proof of a fundamental result in self-similar traffic modeling", ACM SIGCOMM, Computer Communication Review, 1997, pg. 5-23
- [TRA89] Tran-Gia, P., Ahmadi, H. , "Analysis of a Discrete-Time G/D/1/s queueing system with application in packet switching systems", IEEE INFOCOM '88, 1988, pg- 9A.1.10

- [TUC88] Tucker, R.C., "Accurate method for analysis of a packet speech multiplexer with limited delay", IEEE Transaction on Communication, vol. 36, No. 4, 1988, pg. 479
- [TUL98] Tulin, T. and Takahashi, Y., "On the relationship between queue lengths at a random instant and at a departure in the stationary queue with B-Map arrivals", Stochastic Models, vol. 14, 1998, pg. 601-610
- [TUR92] Turner, J.S., "Managing bandwidth in ATM Networks with Bursty traffic", IEEE Network, 1992, pg. 50-58
- [TZE] Tze, D. et al, "A Dynamic Framework for Robust Measurement based Admission Control", available on Internet
- [VEC94] Veciana, G. De, Courcoubetis, C., Walrand, J., "Decoupling bandwidth: a decomposition approach to resource management in networks", Proceedings of the IEEE INFOCOM, 1994, pg. 446-474. <http://walrandpc.eecs.berkeley.edu/Pubs.html>
- [VER89] Verbiest, W., and Pinoo, L., "A variable bit rate video codec for asynchronous transfer mode networks", IEEE Journal on Selected Areas in Communication, vol. 7, 1989, pg. 761-770
- [WAN91] Wang, Y.T., Sengupta, B., "Performance analysis of a Feedback to Congestion Control Policy under non-negligible propagation delay", ACM SIGCOMM '91, pg 968-981
- [WAN92] Wang, Z., Crowcroft, J., "A fluid model approximation to Quantitative Information Feedback in Congestion Control", IEEE IPCCC'92, pg. 0315-0322
- [WHI83] Whitt, W., "The queuing network analyzer", Bell Systems Technical Journal, Nov. 1983, pg. 2779-2813
- [WOO90] Woodruff, G. M., Kositpaiboon, R. "Multimedia traffic management principles for guarantee ATM network performance", IEEE Journal on Selected Areas in Communications, vol 8, No. 3, 1990, pg. 435-446
- [YAM93] Yamada, H., "Cell/packet loss behaviour in a statistical multiplexer with bursty input", Performance Evaluation, vol. 17, 1993, pg. 77-79
- [YAN93] Yang, T., Hui, Li, "Individual Cell Loss Probabilities and Background Effects in ATM Networks", Proceedings of IEEE Globecom, 1993, pg. 1373-1379
- [YAN92] Yang, T., Tsang, D.H.K., "A novel approach to estimating cell loss probability in an ATM multiplexer loaded with homogeneous bursty sources", Proceedings of IEEE Globecom '92, 1992, pg. 515-517

- [YIN91] Yin, N. and Hluchyj, M.G., "A Dynamic Rate Control Mechanism for Source Coded Traffic in a fast Packet Network", IEEE Journal on Selected Areas in Communications, vol 9, September 1991, pg. 1003-1012
- [YE94] Ye, J., Li, S.Q., "Folding Algorithm: A computational Method for finite QBD Process with level-dependent transitions", IEEE Trans. on Communication, vol. 42, No. 2., Feb. 1994, pg. 625-639
- [YIN94] Yin, N., Hluchyj, M.G., "On closed loop rate control for ATM cell relay networks", Proceedings of the IEEE INFOCOM '94, 1994, pg. 99-108
- [ZHA89] Zhang, "A new architecture for packet switching network protocols", PhD thesis, MIT, 1989
- [ZHA93] Zhang, L., et. al., "RSVP: A new Resource Reservation Protocol", IEEE Network 1993
- [ZHU96] Zhu, H., Frost, V., "In-service monitoring for cell loss quality of service violations in ATM networks", IEEE/ACM Trans. Networking, vol. 4, 1996, pg. 240-248

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